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HEAT TRANSFER ANALYSIS OF THE NAHBE PISTON CAP,(U)
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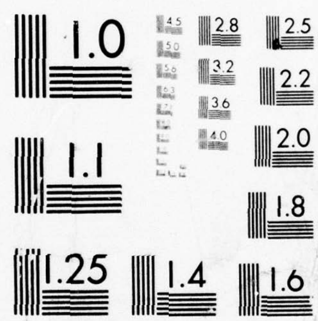

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UNITED STATES NAVAL ACADEMY
DIVISION OF
ENGINEERING AND WEAPONS
ANNAPOLIS, MARYLAND

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HEAT TRANSFER ANALYSIS
OF THE NAHBE PISTON CAP*

Engineering and Weapons Report
EW-11-77

J. Alan Adams

September 1977

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

An analysis of the transient temperature and heat transfer variations in the cap of a modified piston engine (NAHBE) was made. The objective was to estimate the regenerative heating effect due to the cap under various operating conditions.

A finite difference model was developed which accounted for variable heat transfer transfer coefficients and environmental temperatures. An explicit method of solution was obtained by use of a digital computer. (cont.d)

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20.

The parametric study has suggested conditions where a regeneratibe heating effect is significant. It will be more useful when used in conjunction with experimental and future research.

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I ABSTRACT

The piston modification for the NAHBE (Naval Academy Heat Balance Engine) consists of a cap which extends above the standard piston used in an Otto cycle.[1] This modification, together with an auxiliary air inlet between the intake valve and fuel supply, has been shown to produce a significant influence on combustion within the cylinder. One theory to explain this behavior is that time dependent heat addition may be affected in a field of pressure waves sustained by pressure exchange between the two chambers.[2]

The objectives of this study were twofold. First, the transient convective boundary conditions on the cap were studied to estimate what probable values of heat transfer coefficients and environmental temperatures were creating the temperature behavior within the cap material. Second, the heat transfer to and from the cap and the surrounding gases within the cylinder was calculated.

A mathematical model was developed which predicts the transient, periodic temperature response of the cap as a function of time dependent convective boundary conditions. Total heat transfer to and from the cap, along with instantaneous heat flux rates, were obtained for various operating conditions. Two sets of convective heat transfer coefficients were used, a set of low values and a set of high values. The actual convective heat transfer process is not fully understood, but these initial parametric results give an indication of the regenerative effect due to heat transfer from the cap to the air-fuel mixture which can be expected under various conditions.

II FOREWORD

The aluminum cap on the piston head within a NAHBE engine undergoes a complex thermal response during the engine operation. The complex surface-gas interaction during combustion is not fully understood and thus the resistance to heat transfer by convection and radiation between the gas and solid cap can only be estimated. However, parametric studies using ranges of value which include the suspected actual values can give a useful insight into the thermal process.

In this study the top, bottom, and edge surfaces of the cap are exposed to three different environmental conditions through computer simulation. The three environmental temperatures and the three surface heat transfer coefficients can change independently during each stroke of the 4-stroke engine. The speed of the engines can be varied and results are given in Section IV for speeds between 500 and 5,000 rpm. The simulation model gives the temperature of the cap as a function of radius and time, the instantaneous surface heat transfer rates, and the total heat transfer to and from the cap during a specified time interval. A listing of the computer programs used in this simulation appears in Appendix A.

III MATHEMATICAL MODEL

The cap is modeled as a radial fin of constant thickness (δ). The inside base radius r_i = 1.0 in., the outside radius r_o = 1.75 in., and the thickness (initially) δ = 0.01 in. The temperature is assumed to be uniform across the fin thickness at a given radius and time. The differential equation for this transient, one-dimensional heat conduction problem is given by:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{1}{k\,\delta}\left[h_T(T-T_{\infty,T}) - h_B\,(T-T_{\infty,B}) - h_E\,(T-T_{\infty,E})\right] = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Here, h_T, h_B, and h_E are the time dependent heat transfer coefficients on the top, bottom, and edge surfaces respectively. Likewise, $T_{\infty,T}$, $T_{\infty,B}$, and $T_{\infty,E}$ are the time dependent environmental gas temperatures for the three respective regions.

The boundary conditions are as follows:

at $r = r_i$, $T = T_w = 500$ F

at $r = r_o$, $-\ k\left(\frac{\partial T}{\partial r}\right)_{r=r_o} = h_E\ (T-T_{\infty,E})$

The initial condition is:

at $t = o$, $T = 500$; $r_i \leq r \leq r_o$.

An explicit, finite difference model is used to approximate the above partial differential equation. A grid consisting of thirteen nodes (Δr = 1/16 in.) is chosen to approximate the fin. Application of the first law of thermodynamics to this grid gives the following set of explicit, finite difference equations.

NODE	EQUATION
1	$T_i^{n+1} = 500$
2-12	$T_i^{n+1} = T_i^n \, [1-(\frac{\alpha\Delta t}{k\delta})(h_B + h_T) - 2\,(\frac{\alpha\Delta t}{\Delta_r^2})]$ $+ (\frac{\alpha\Delta t}{\Delta r^2})\,[T_{i+1}^n + T_{i-1}^n]$ $+ \frac{1}{2}\,(\frac{\alpha\Delta t}{\Delta r^2})(\frac{\Delta r}{r_i})\,[T_{i+1}^n - T_{i-1}^n]$ $+ (\frac{\alpha\Delta t}{k\delta})\,[h_T\, T_{\infty,T} + h_B\, T_{\infty,B}]$
13	$T_i^{n+1} = (1/G)\,[T_{i-1}^n + (\frac{h_E\Delta r}{k})\,T_{\infty,E}]$ $+ [1 - \frac{1 + (h_E\Delta r/k)}{G}]\,T_i^n$

Here T_i^n represents the temperature at node i ($1 \leq i \leq 13$) for a given time, t = n. T_i^{n+1} represents the temperature at node i for the time $t + \Delta t = n + 1$. The constant $G = (\Delta x)^2/\alpha\Delta t$.

Numerical stability requires that the time step Δt be chosen such that:

$$(\frac{\alpha\Delta t}{k\delta})\,(h_B + h_T) + 2(\frac{\alpha\Delta t}{\Delta r^2}) < 1.0$$

For an aluminum fin with h ~ 50 BTU/hr-ft^2- F, this requires $\Delta t < 0.01$ sec. In the NAHBE analysis this does not impose a limitation since much smaller time increments are used to account for the rapid variations in h and T_∞ during the 4-stroke cycle.

The assumed time variation of $T_{\infty,T}$, the environmental gas temperature above the fin, is shown in Figure 1 for five complete cycles at an

RPM of 1500. An assumed maximum temperature of 4500 F occurs at the end of the expansion stroke. Intake air at 70 F and exhaust at 1218 F is used. The gas temperature variation beneath the cap is given in Figure 2, and the assumed gas temperature variation at the edge is shown in Figure 3. These functions can be easily changed as a better understanding of the actual process is obtained.

The assumed variation of the convective heat transfer coefficients with time are estimated based upon the normal range of expected valves for turbulent flow of a gas over a surface. In order to establish a benchmark result a low value of h = 5 BTU/hr-ft^2-F was used for h_T, h_B, and h_E during intake. A value of 10 BTU/hr-ft^2-F was used during the other three strokes of compression, expansion, and exhaust. Table A gives the values used.

STROKE	TOP	EDGE	BOTTOM
EXHAUST	10	10	10
INTAKE	5	5	5
COMPRESSION	10	10	10
EXPANSION	10	10	10

Table A: Heat Transfer Coefficients for Benchmark Runs

All values can be assigned independently as more knowledge is gained. The benchmark runs all used the above values of the convective heat transfer coefficients, as well as the previously described environmental temperature variations.

A second set of computer solutions was obtained using values of convective heat transfer coefficients near those reported in Reference 3

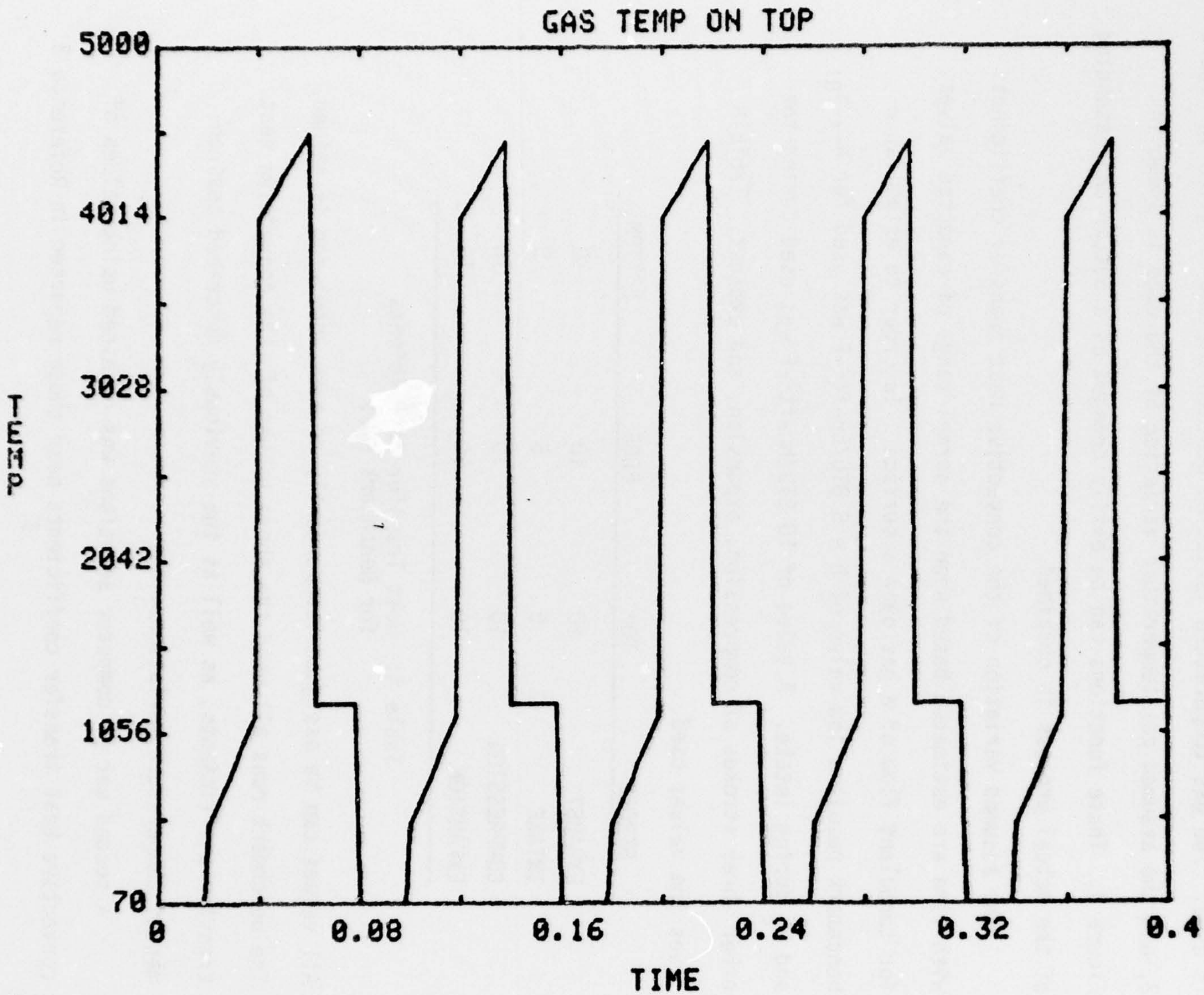

FIGURE 1: Variation of $T_{\infty,T}$ with time

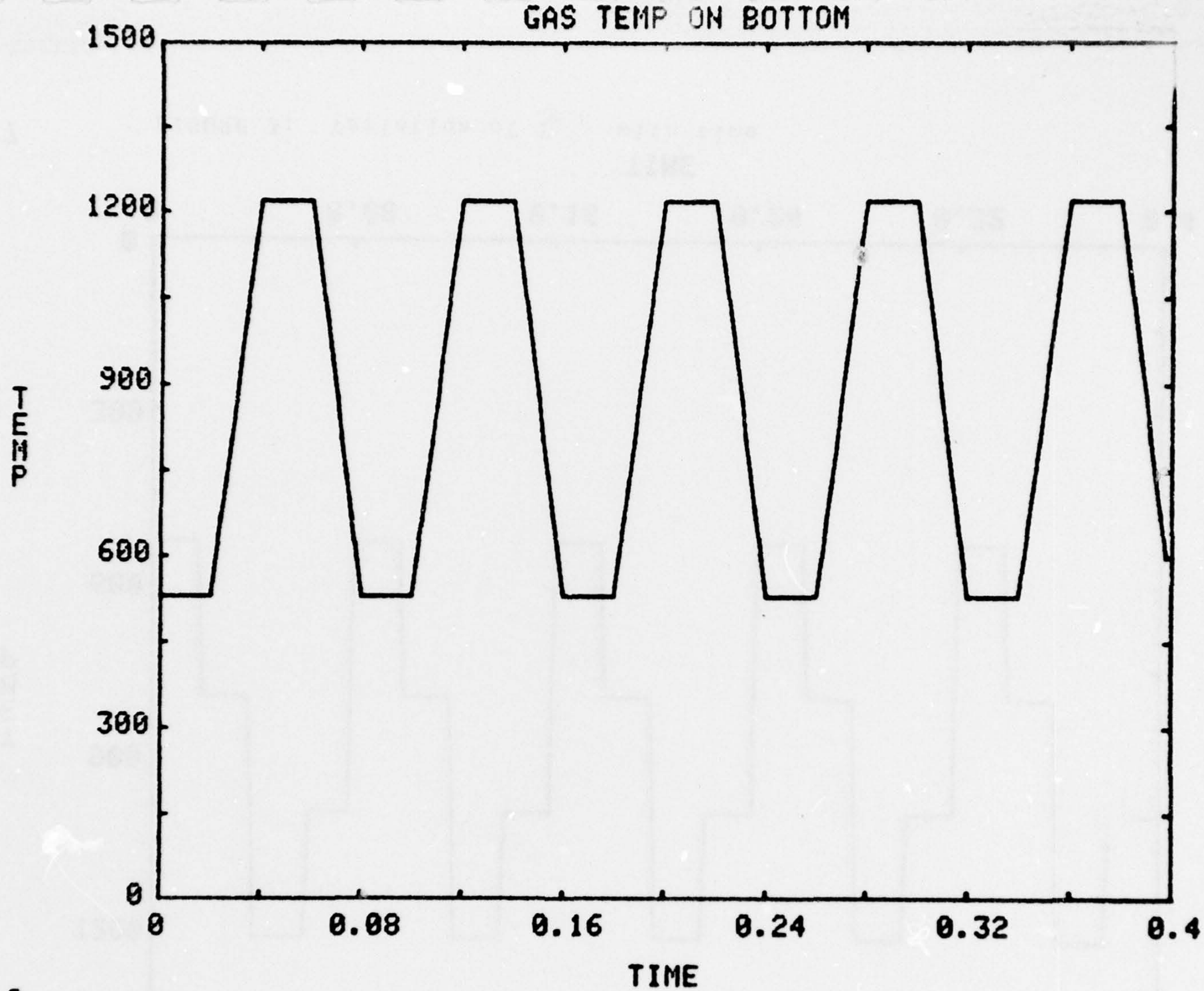

FIGURE 2: Variation of $T_{\infty,\beta}$ with time

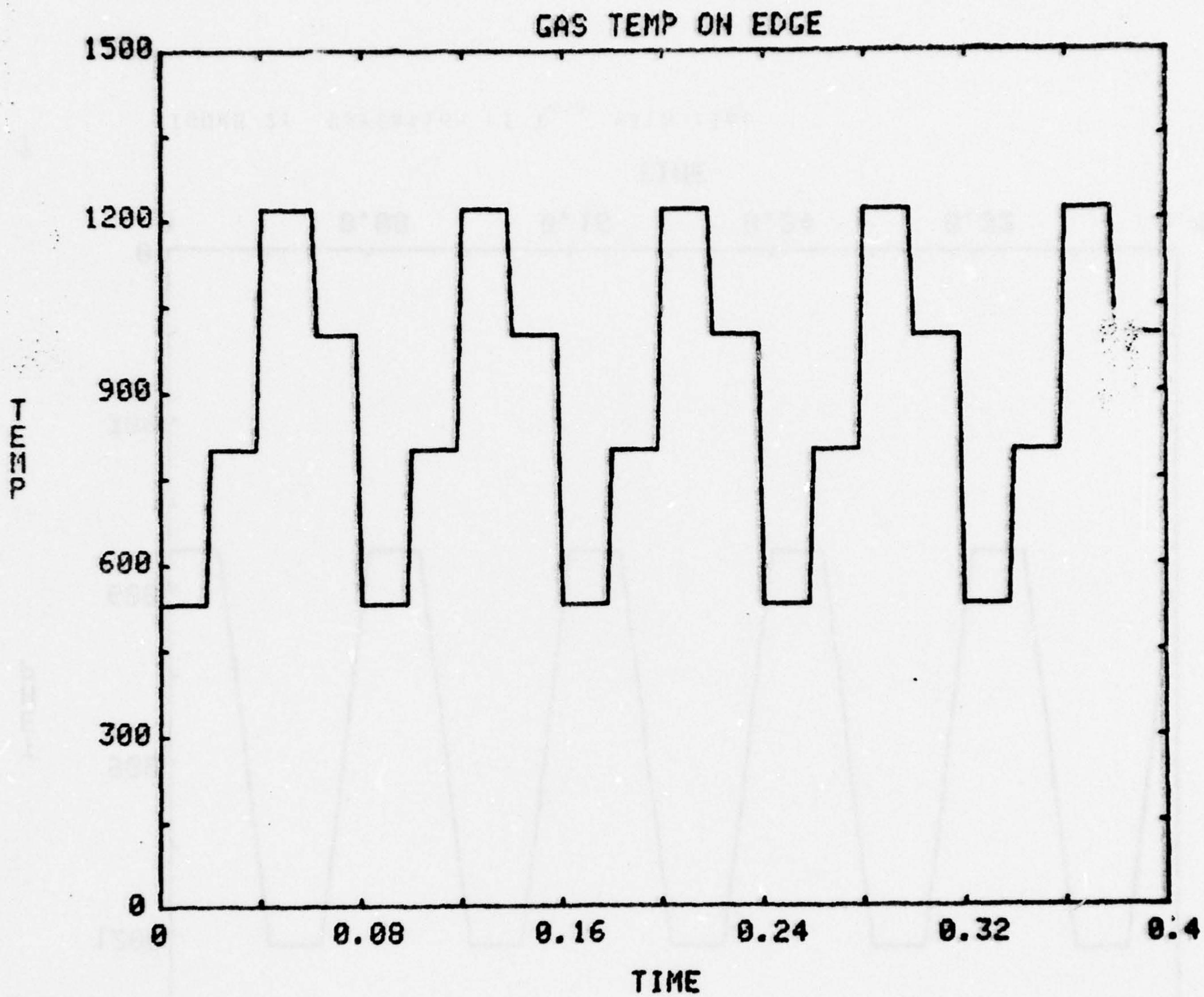

FIGURE 3: Variation of $T_{g,E}$ with time

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based upon results obtained by Eaton Corporation for exhaust valves. These values are tabulated in Table B.

STROKE	TOP	EDGE	BOTTOM
EXHAUST	60	60	40
INTAKE	80	40	30
COMPRESSION	60	60	40
EXPANSION	60	60	40

Table B: Heat Transfer Coefficients for Eaton Runs

IV DISCUSSION OF RESULTS

A. Benchmark Series with Parametric Variations

Figure 4 shows the temperature response of an aluminum cap as a function of time for nodes 3, 5, 7, 9, 11, and 13 using benchmark conditions for h and a speed of 500 rpm. The initial temperature is 500 F throughout the fin and a quasi-steady state condition is reached after 8 seconds. As can be seen, node 3 reaches a mean temperature of 570 F with oscillation $\pm$ 8^{0}F. Node 5 reaches 625 F, node 7 reaches 665 F, and node 9 reaches 685 F. Nodes 11 and 13 are affected by the edge convective boundary condition and behave in approximately the same manner. The numerical values in Table 1 show that the edge temperature (node 13) oscillates between 697 F and 705 F during the time interval $8.7 \le t \le 10$. Table 1 also shows that the total heat transfer from the cap to the gas during the initial 10 second interval is 0.1093 BTU and the total heat transfer from the gas to the cap in 1.73 BTU during the same interval.

Figure 5 shows the transient response for the benchmark run at 1500 RPM, and Table 2 gives the numerical values after quasi-steady conditions are achieved. The mean temperatures obtained at 1500 RPM are very close to those at 500 RPM. However, the temperature oscillations are much smaller at 1500 RPM as expected. The total heat transfer from the cap to the gas is only slightly higher at 1500 RPM, e.g. 0.1118 vice 0.1093.

The 5000 RPM benchmark results are given in Figure 6 and Table 3. Note the scale change for the ordinate in Figure 6. Most noticeable are that the nodal temperatures have increased slightly and the nature of the oscillations have radically changed. The maximum temperature reached

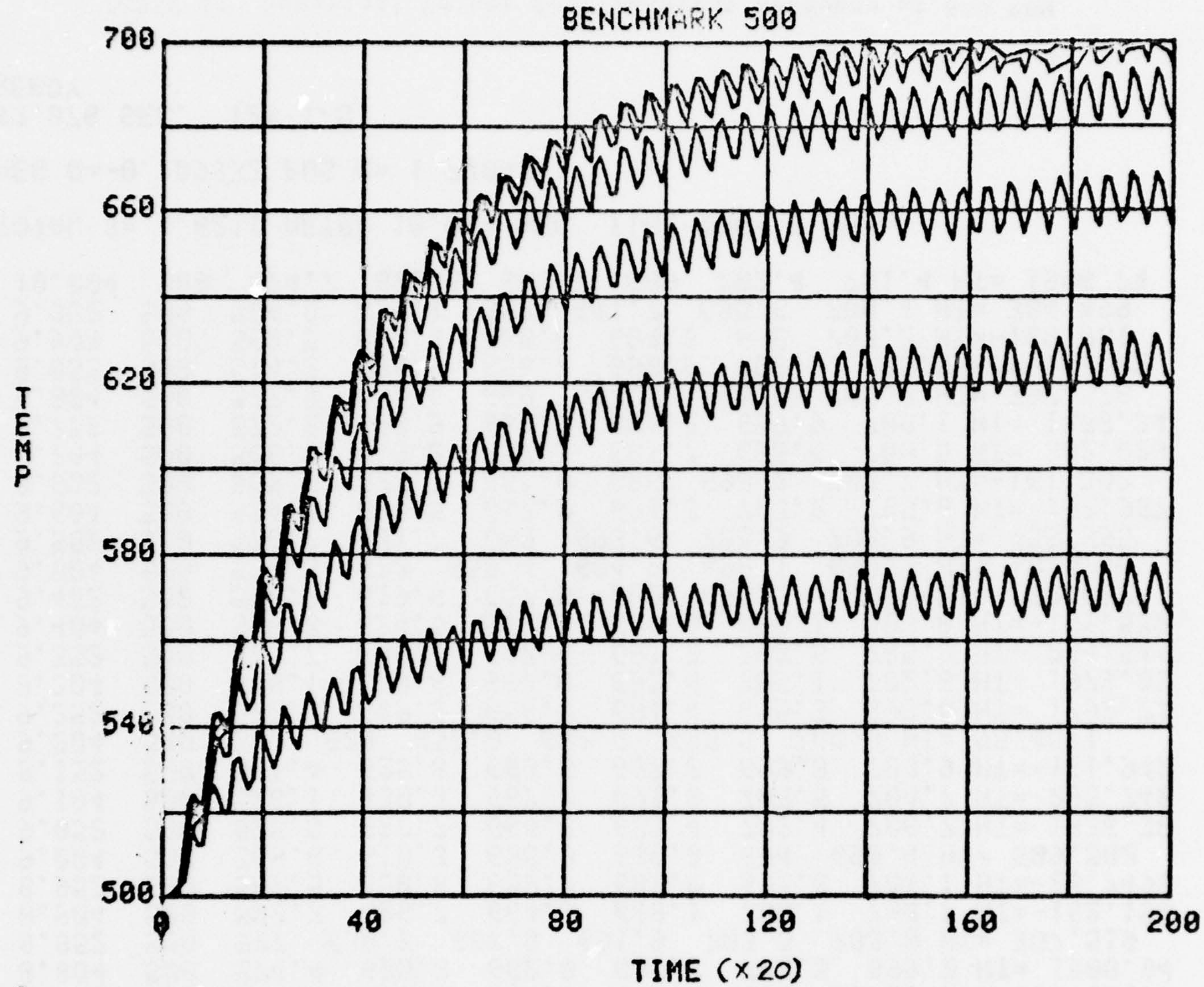

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FIGURE : Transient Response at 500 RPM

(–)	(1)	(3)	(5)	(7)	(9)	(11)	(13)	
00.1	703.2	HT=-162.188						
8.7	500	567.8	621.1	658.7	682.8	695.6	700.8	HT=-160.055
8.752	500	565.9	617.4	654.8	679.2	693	697.8	HT= 386.569
8.804	500	574.4	625.8	662.8	687.3	700.5	699.2	HT= 1908.04
8.852	500	577	630.7	667.8	691.9	703.5	703.8	HT= 357.519
8.904	500	572.3	626.7	664.3	688.1	700.1	703.7	HT=-162.156
8.952	500	567.5	620.8	658.7	682.8	695.8	701.1	HT=-28.7499
9.004	500	566.6	618.3	655.8	680.3	694	698.4	HT= 509.502
9.052	500	575.8	627.7	664.8	689.4	702.4	700.7	HT= 1936.38
9.104	500	576.1	630.3	667.7	691.8	703.5	704.7	HT= 295.748
9.152	500	571.4	625.8	663.6	687.6	699.8	703.9	HT=-161.942
9.204	500	567	620	657.8	682.2	695.5	700.9	HT= 95.4571
9.252	500	567.7	619.3	656.8	681.4	695.3	698.6	HT= 1792.71
9.304	500	578.1	630.6	667.8	692.4	705.1	702.6	HT= 1979.03
9.352	500	575.7	630.1	667.6	691.7	703.5	705.2	HT= 254.544
9.404	500	570.3	624.5	662.5	686.6	699.1	703.8	HT=-161.538
9.452	500	566.9	619.6	657.5	682	695.5	700.8	HT= 178.077
9.504	500	570.5	622	659.4	684.2	698.1	699.3	HT= 1835.49
9.552	500	578.7	631.7	669	693.6	705.9	703.9	HT= 459.395
9.604	500	574.9	629.5	667.2	691.3	703.2	705.6	HT= 192.967
9.652	500	569.6	623.7	661.8	686	698.7	703.7	HT=-161.302
9.704	500	566.8	619.2	657.1	681.7	695.4	700.5	HT= 302.028
9.752	500	572.2	623.9	661.3	686.2	699.9	700.1	HT= 1863.94
9.804	500	577.9	631.6	669	693.6	705.6	705.2	HT= 397.578
9.852	500	574.2	628.9	666.7	690.9	702.9	705.8	HT=-163.22
9.904	500	568.5	622.4	660.6	684.9	698	703.2	HT=-160.881
9.952	500	566.8	619.1	657	681.7	695.6	700.4	HT= 384.499
10.004	500	574.7	626.6	664.1	689	702.4	701.4	HT= 1906.74

TOTAL Q= 1.6211 AFTER 10.004 SEC TIME INC= 0.004

NEG Q=-0.109333 POS Q= 1.73043

93.078 SEC. 124 I/O
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TABLE 1: Numerical Values for Transient Response at 500 RPM

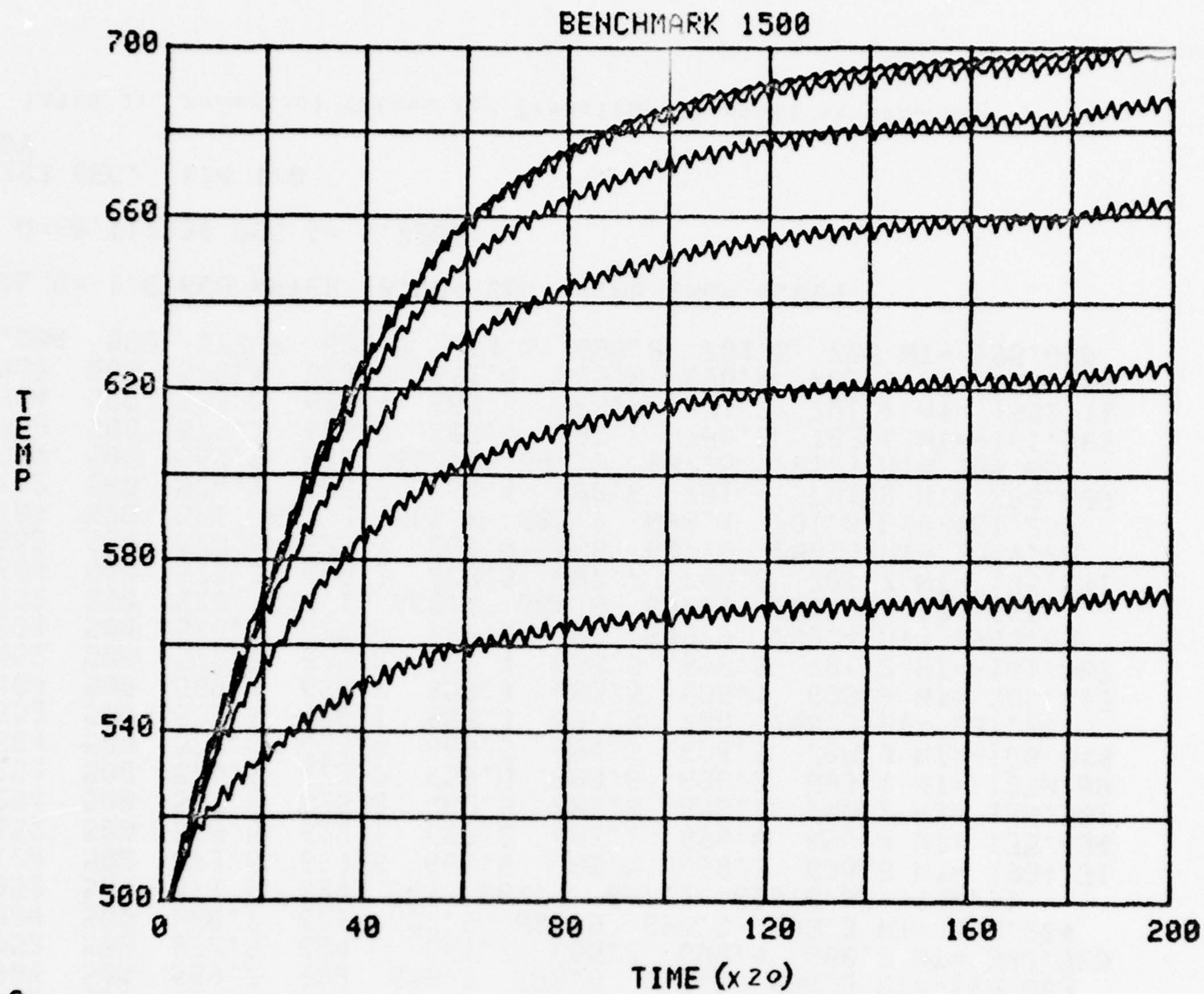

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FIGURE 5: Transient Response at 1500 RPM

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(t)  (1)  (3)  (5)  (7)  (9)  (11)  (13)  q
00  570.7  622.9  659.7  683.6  696  698.8 HT=-160.4
 8.7  500  571.6  623.4  660.3  684.2  697  697.8 HT= 1940.42
 8.752  500  568.6  620.8  657.7  681.6  694.1  698.2 HT= 137.233
 8.804  500  571.8  623.9  660.7  684.7  697.1  698.9 HT=-160.816
 8.852  500  569.1  621  657.9  681.9  694.7  697.5 HT= 1852.33
 9.904  500  569.7  622  658.9  682.8  695.3  698.8 HT=-160.083
 8.952  500  572.9  624.8  661.7  685.7  698.4  698.7 HT= 382.925
 9.004  500  568.7  621  657.9  681.9  694.5  698.3 HT= 508.204
 9.052  500  571.6  624  661  684.9  697.3  699.6 HT=-160.91
 9.104  500  572.5  624.6  661.6  685.5  698.3  698.8 HT= 1984.31
 9.152  500  569.5  622.1  659.2  683.1  695.6  699.4 HT= 135.894
 9.204  500  572.8  625.2  662.3  686.3  698.7  700.2 HT= 196.901
 9.252  500  570.1  622.5  659.6  683.6  696.3  699.1 HT= 1850.89
 9.304  500  570.5  623.4  660.5  684.6  696.9  700.3 HT=-160.744
 9.352  500  573.6  626.1  663.3  687.3  700  700.3 HT= 381.73
 9.404  500  569.4  622.2  659.4  683.5  696.1  699.9 HT= 506.813
 9.452  500  572.3  625.2  662.4  686.5  698.9  701.2 HT=-161.507
 9.504  500  573.1  625.6  662.9  687  699.9  700.4 HT= 1983.09
 9.552  500  570  623.1  660.4  684.6  697.1  701 HT= 134.611
 9.604  500  573.3  626.2  663.5  687.7  700.2  701.7 HT= 195.891
 9.652  500  570.6  623.4  660.8  685  697.8  700.6 HT= 1849.78
 9.704  500  571  624.2  661.7  685.9  698.4  701.8 HT=-161.259
 9.752  500  574.1  626.9  664.4  688.6  701.4  701.8 HT= 380.809
 9.804  500  569.9  623  660.5  684.7  697.5  701.3 HT= 505.681
 9.852  500  572.7  625.9  663.4  687.7  700.3  702.6 HT=-161.981
 9.904  500  573.5  626.4  663.9  688.2  701.2  701.7 HT= 1982.11
 9.952  500  570.4  623.8  661.4  685.8  698.4  702.3 HT= 133.554
 10.004  500  573.7  626.9  664.5  688.8  701.5  703 HT= 195.099

TOTAL Q= 1.61668 AFTER 10.004 SEC  TIME INC= 0.004

NEG Q=-0.111836 POS Q= 1.72851

92.393 SEC.  114 I/O
READY
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TABLE 2: Numerical Values for Transient Response at 1500 RPM

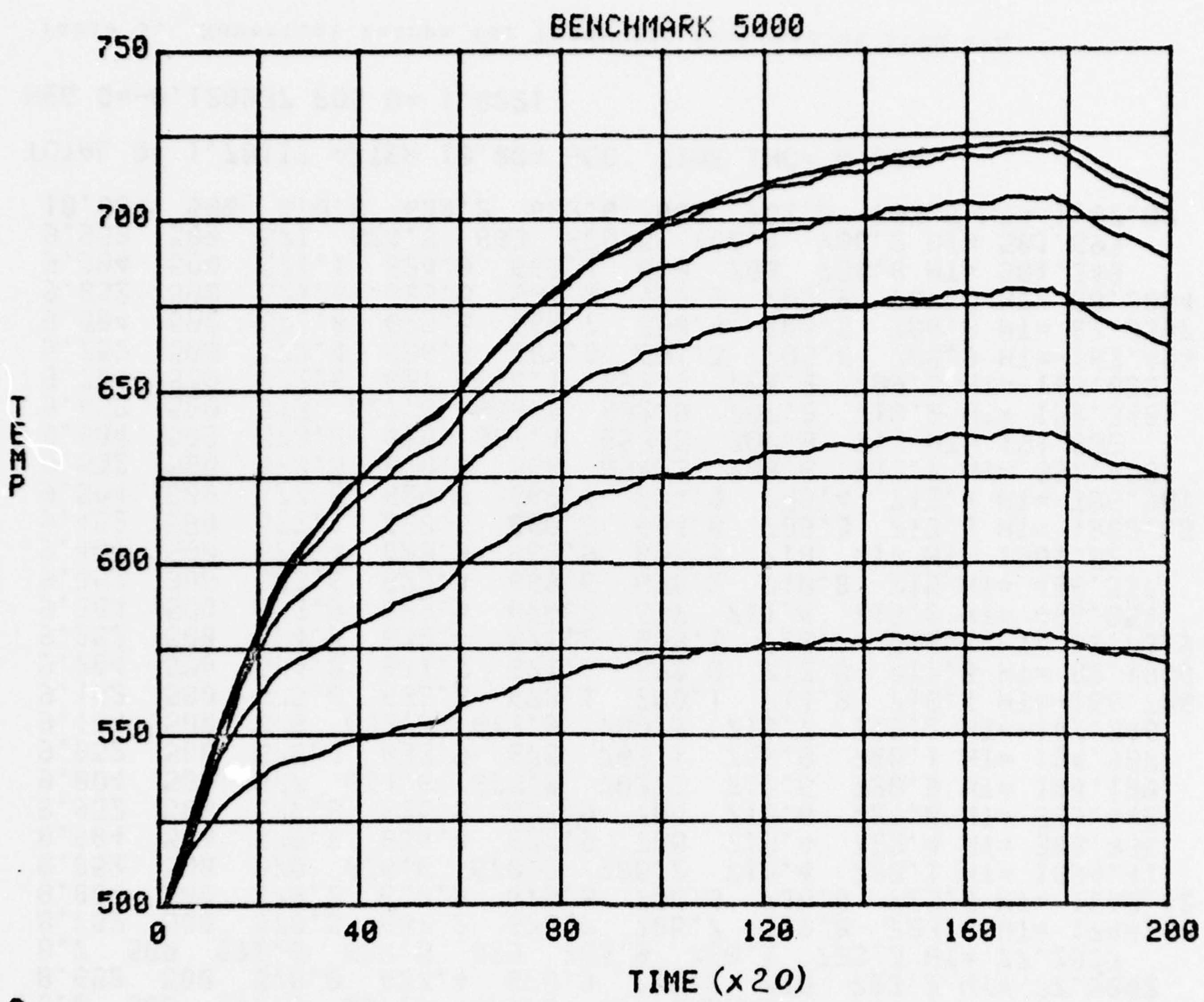

FIGURE 6: Transient Response at 5000 RPM

8.552	500	579.3	637.8	679.3	[illegible]	[illegible]	[illegible]	HT=-169.062
8.6	500	579.3	637.9	679.3	706.2	720.4	723.1	HT=-169.096
8.652	500	578.8	637.4	678.9	705.8	720	723.2	HT= 77.5202
8.7	500	578.8	637.5	679	705.9	720.1	723.3	HT= 77.3923
8.752	500	578.5	637.2	678.7	705.7	719.8	723.3	HT= 1744.17
8.804	500	579.2	637.9	679.5	706.5	720.6	723.3	HT= 1893.42
8.852	500	578	636.6	678.2	705.2	719.4	723.1	HT= 1894.41
8.904	500	578.2	636.4	677.9	705	719.4	722.4	HT= 388.836
8.952	500	577.6	635.5	676.9	704	718.6	721.6	HT= 389.612
9.004	500	577	634.6	675.9	702.9	717.6	720.9	HT= 184.188
9.052	500	576.6	633.9	675	702.1	716.8	720.1	HT= 184.907
9.104	500	576	632.9	673.9	700.9	715.7	719.2	HT=-167.046
9.152	500	575.6	632.3	673.1	700.1	714.8	718.4	HT=-166.726
9.204	500	574.8	631.3	671.9	698.8	713.5	717.5	HT= 82.1946
9.252	500	574.5	630.7	671.2	698.1	712.7	716.7	HT= 82.7319
9.304	500	573.9	629.9	670.3	697	711.6	715.8	HT= 496.054
9.352	500	573.6	629.4	669.6	696.2	710.8	715	HT= 496.557
9.404	500	573.4	628.9	668.9	695.5	710	714	HT= 1901.62
9.452	500	573.1	628.5	668.3	694.8	709.3	713.3	HT= 1902.13
9.504	500	573.8	628.9	668.6	694.9	709.4	712.4	HT= 396.961
9.552	500	573.5	628.5	668	694.3	708.6	711.7	HT= 397.498
9.604	500	573.2	628	667.4	693.5	707.8	711	HT= 191.832
9.652	500	573	627.6	666.9	692.9	707.2	710.2	HT= 192.342
9.704	500	572.6	627	666.1	692.1	706.2	709.5	HT=-163.635
9.752	500	572.4	626.6	665.6	691.5	705.6	708.9	HT=-163.408
9.804	500	571.8	625.8	664.7	690.4	704.5	708.2	HT= 87.8397
9.852	500	571.6	625.5	664.2	689.9	703.9	707.5	HT= 88.2054
9.904	500	571.1	624.9	663.5	689	703	706.8	HT= 501.349
9.952	500	571	624.5	663	688.5	702.4	706.2	HT= 501.693
10.004	500	570.8	624.2	662.6	688	701.8	705.5	HT= 1907.09

TOTAL Q= 1.70117 AFTER 10.004 SEC   TIME INC= 0.004

NEG Q=-0.120937 POS Q= 1.8221

Table 3: Numerical Values for Transient Response at 5000 RPM

at the edge of the fin is 25 F above the lower RPM valves. The heat transfer to the gas from the cap is now 0.1209 BTU and the heat transfer to the cap is 1.822 BTU during the first ten seconds of run time. When integration is carried out between 10 sec $\leq$ to $\leq$ 20 seconds, with the cap temperatures relatively constant over the entire interval, these heat transfer results are increased by approximately 20% during 10 sec $\leq t \leq 20$ sec.

To show the parametric effect of changing the convective heat transfer coefficient during intake, a change was made to the benchmark values. Figure 7 shows the effect of increasing the convective heat transfer coefficients during intake on the top, bottom, and edge of the cap by a factor of 5, from 5 BTU/hr-ft^2-F to 25 BTU/hr-ft^2-F. All other variables remain the same as the benchmark runs. At 500 RPM, the results in Figure 7, compared with Figure 4, show that the temperatures along the cap are reduced by approximately 40F. Comparison of Table 4 with Table 1 shows that the heat transfer from the cap to the gas is now 0.4809 BTU rather than 0.1093 BTU, an increase by a factor of 4.4. The heat transfer to the cap is also slightly higher, 1.775 BTU vice 1.730 BTU. It must be remembered that these results all depend upon the somewhat arbitrary assumption of the gas temperatures shown in Figures 1-3.

Figure 8 and Table 5 show the same effect for the same high intake h values at 1500 RPM. Again, a 40 F temperature drop is observed and the same increase in heat transfer to the gas is obtained.

Figure 9 and Table 6 are included to show that approximately the same effect can be obtained by increasing only the convective heat transfer

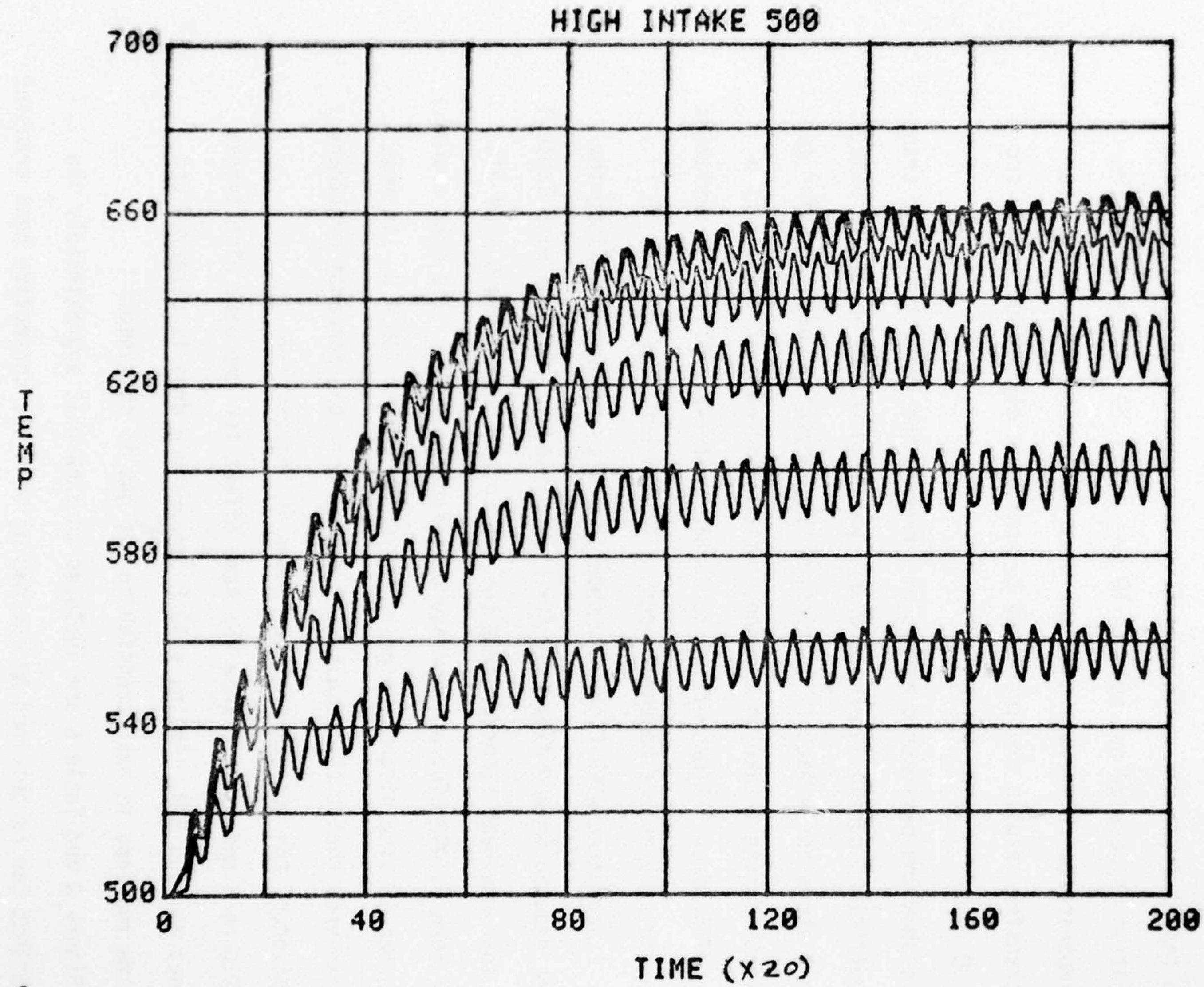

FIGURE 7: Transient Response at 500 RPM for High h Intake Values

8.852	500	563.5	606.3	635.4	654.3	663.2	662.5	HT= 386.249
8.904	500	558.3	601.9	631.5	650.1	659.1	662.9	HT=-737.357
8.952	500	551.4	593.2	622.8	641.4	651.9	658.4	HT= 2.7117
9.004	500	551.8	591.9	621.1	640.4	651.6	655.4	HT= 539.961
9.052	500	561.7	602.4	631.4	650.8	661.1	658.6	HT= 1965.92
9.104	500	562.7	606.1	635.4	654.4	663.2	663.4	HT= 324.367
9.152	500	556.9	600.4	630.2	648.8	658.1	662.8	HT=-734.885
9.204	500	551.2	592.5	622.1	641	652	657.7	HT= 126.774
9.252	500	552.9	593	622.2	641.6	653	655.7	HT= 1823.1
9.304	500	564.1	605.5	634.6	653.9	663.8	660.6	HT= 2008.44
9.352	500	562.3	605.9	635.5	654.4	663.3	664	HT= 283.112
9.404	500	555	598.2	628	646.7	656.4	662	HT=-730.946
9.452	500	551.2	592.3	621.9	641	652.1	657.5	HT= 209.322
9.504	500	555.8	595.9	625.1	644.6	655.9	656.5	HT= 1865.74
9.552	500	564.8	606.6	635.8	655.2	664.8	661.9	HT= 488.744
9.604	500	561.7	605.5	635.3	654.1	663.1	664.4	HT= 221.406
9.652	500	553.9	596.8	626.7	645.4	655.4	661.5	HT=-728.455
9.704	500	551.3	592.1	621.7	640.9	652.3	657.1	HT= 333.117
9.752	500	557.7	597.9	627.1	646.7	657.8	657.3	HT= 1894.11
9.804	500	564.1	606.8	636.1	655.4	664.6	663.3	HT= 426.785
9.852	500	560.7	604.6	634.5	653.3	662.4	664.7	HT=-743.62
9.904	500	552.3	594.7	624.5	643.3	653.7	660.4	HT=-724.52
9.952	500	551.6	592.2	621.7	641.1	652.5	656.9	HT= 415.503
10.004	500	560.3	600.9	630.1	649.7	660.4	658.7	HT= 1936.74

TOTAL Q= 1.29444 AFTER 10.004 SEC TIME INC= 0.004

NEG Q=-0.48088 POS Q= 1.77532

90.942 SEC. 116 I/O
READY

TABLE 4: Numerical Values for Transient Response at 500 RPM for High h Intake Values

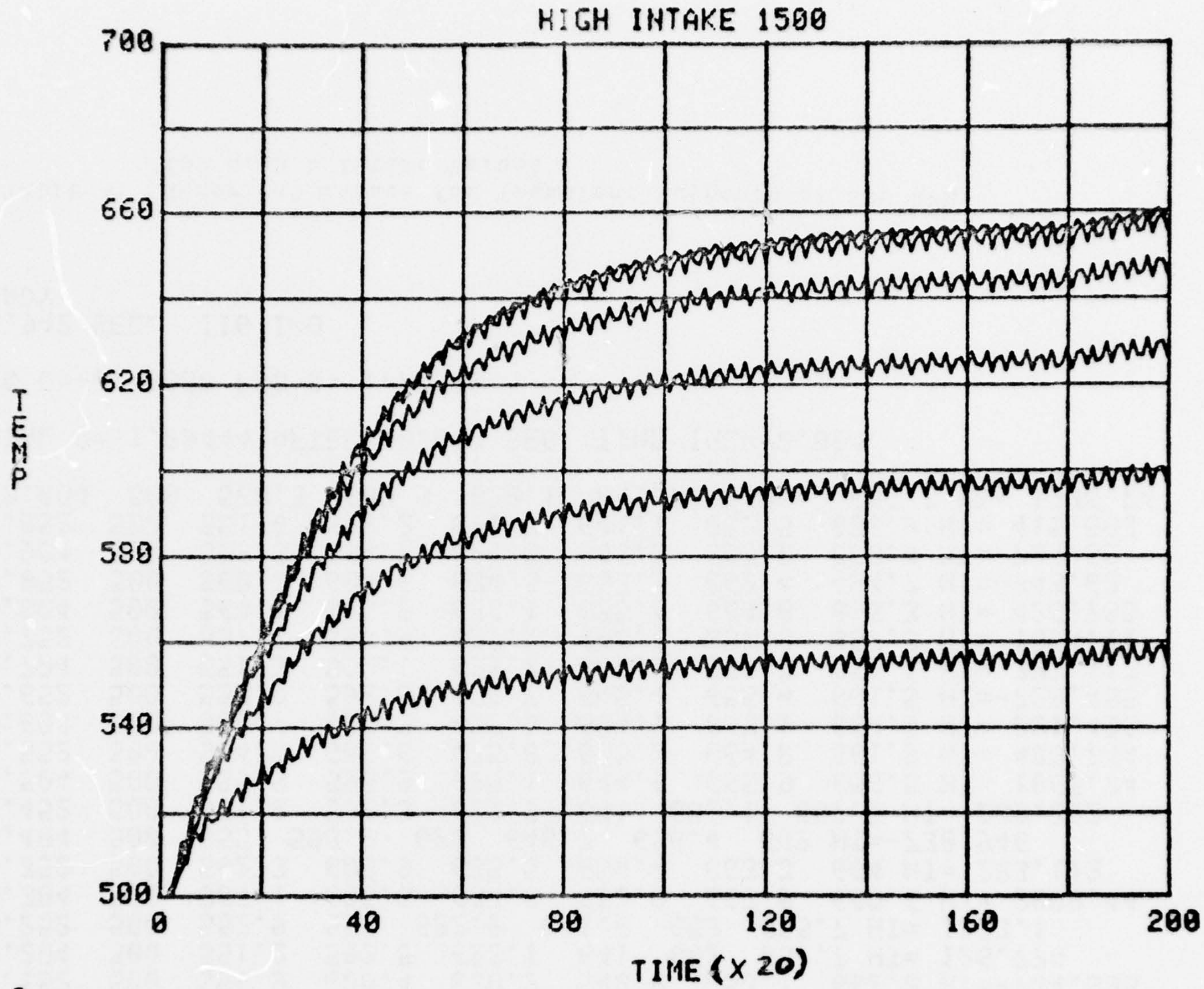

FIGURE 8: Transient Response at 1500 RPM for High h Intake Values

4 HT=-730.511

	(1)	(3)	(5)	(7)	(9)	(11)	(13)	
8.852	500	554.3	595	623.8	642.5	652.7	655.3	HT= 1882.36
8.904	500	554.7	595.8	624.5	643.2	652.9	656.8	HT=-723.983
8.952	500	558.6	599.3	628.1	646.8	656.9	656.7	HT= 412.588
9.004	500	553.9	594.8	623.7	642.3	652.3	655.9	HT= 538.413
9.052	500	557.3	598.6	627.4	646.1	655.8	657.8	HT=-729.75
9.104	500	558	598.9	627.8	646.4	656.6	656.6	HT= 2014.14
9.152	500	554.4	595.7	624.6	643.2	652.9	657.1	HT= 166.36
9.204	500	558.6	600	628.9	647.7	657.4	659.2	HT= 226.389
9.252	500	555.4	596.5	625.5	644.2	654.3	656.6	HT= 1881.
9.304	500	555.5	597.1	626.1	644.8	654.4	658.2	HT=-727.084
9.352	500	559.3	600.6	629.6	648.3	658.4	658.1	HT= 411.473
9.404	500	554.6	596	625.1	643.8	653.7	657.4	HT= 537.108
9.452	500	558	599.7	628.8	647.6	657.3	659.2	HT=-732.518
9.504	500	558.6	599.9	629	647.8	658	659	HT= 2013.02
9.552	500	554.9	596.7	625.8	644.5	654.3	658.5	HT= 165.169
9.604	500	559.1	600.9	630.1	648.9	658.8	659.6	HT= 225.471
9.652	500	555.9	597.4	626.6	645.4	655.6	658	HT= 1879.97
9.704	500	556	597.9	627.1	646	655.7	659.5	HT=-729.417
9.752	500	559.7	601.3	630.6	649.5	659.7	659.5	HT= 410.645
9.804	500	555	596.7	626	644.9	655	658.7	HT= 536.072
9.852	500	558.4	600.4	629.7	648.7	658.5	660.5	HT=-734.648
9.904	500	559	600.6	629.9	648.9	659.2	659.2	HT= 2012.14
9.952	500	555.3	597.3	626.7	645.6	655.5	659.7	HT= 164.21
10.004	500	559.5	601.5	630.9	650	659.9	660.8	HT= 224.773

TOTAL Q= 1.29141 AFTER 10.004 SEC TIME INC= 0.004

NEG Q=-0.482173 POS Q= 1.77358

90.227 SEC. 114 I/O
READY

TABLE 5: Numerical Values for Transient Response at 1500 RPM for High H INtake Values

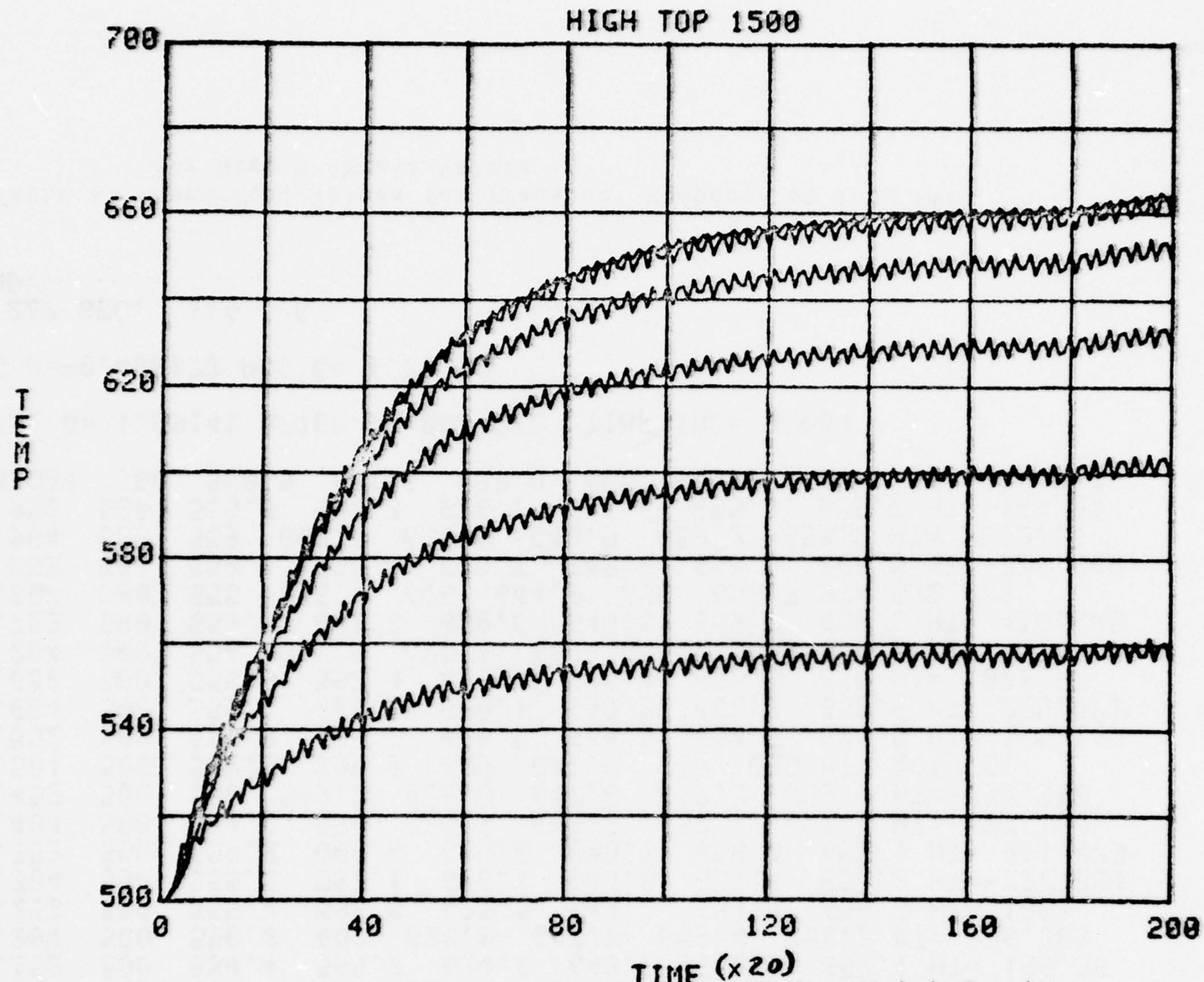

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FIGURE 9: Transient Response at 1500 RPM for High h Intake on Top Cap Surface Only

	(1)	(3)	(5)	(7)	(9)	(11)	(13)	q
58.7	667.7	666.8	HT= 382.997					
8.904	500	559.8	604.8	635.4	654.5	663.8	667.3	HT=-653.454
8.952	500	553	596.4	627.1	646.4	657.1	663.2	HT=-0.959235
9.004	500	553.4	594.9	625.2	645.1	656.6	660.3	HT= 536.411
9.052	500	563.3	605.3	635.3	655.3	665.9	663.2	HT= 1962.53
9.104	500	564.3	608.9	639.2	658.7	667.7	667.8	HT= 321.116
9.152	500	558.5	603.4	634.1	653.4	662.9	667.3	HT=-652.123
9.204	500	552.8	595.7	626.4	646	657.1	662.7	HT= 123.1
9.252	500	554.6	596	626.3	646.4	658	660.6	HT= 1819.55
9.304	500	565.7	608.4	638.5	658.5	668.6	665.1	HT= 2005.05
9.352	500	563.9	608.7	639.2	658.7	667.8	668.4	HT= 279.853
9.404	500	556.6	601.3	632.2	651.5	661.4	666.7	HT=-649.962
9.452	500	552.9	595.5	626.2	645.9	657.3	662.5	HT= 205.639
9.504	500	557.5	598.9	629.2	649.3	660.9	661.3	HT= 1862.19
9.552	500	566.4	609.6	639.7	659.7	669.6	666.5	HT= 485.353
9.604	500	563.2	608.3	639	658.5	667.7	668.8	HT= 218.154
9.652	500	555.5	600	630.9	650.3	660.5	666.3	HT=-648.61
9.704	500	553	595.3	626	645.9	657.4	662.1	HT= 329.436
9.752	500	559.4	600.9	631.2	651.4	662.8	662.1	HT= 1890.57
9.804	500	565.7	609.7	640	659.9	669.3	667.8	HT= 423.402
9.852	500	562.3	607.5	638.3	657.8	667.1	669.1	HT=-657.198
9.904	500	554	597.9	628.9	648.4	659	665.4	HT=-646.438
9.952	500	553.3	595.3	626	646	657.6	662	HT= 411.818
10.004	500	562	603.9	634.2	654.4	665.4	663.5	HT= 1933.22

TOTAL Q= 1.33527 AFTER 10.004 SEC TIME INC= 0.004

NEG Q=-0.436346 POS Q= 1.77162

TABLE 6: Numerical Values for Transient Response at 1500 RPM for High h Intake on Top Cap Surface Only

coefficient on the top surface of the cap to 25 BTU/hr-ft^{2}F, and leaving all others at the benchmark values. This can be seen by comparing Figures 8-9, and Tables 5-6. In this case the total heat transfer from the cap to the gas is 0.4363 BTU during the first ten seconds.

Instantaneous heat transfer values are also shown in the last column of the previous six tables. When the base cap temperature is kept at 500 F, as assumed, the heat transfer is predominantly positive (into the cap from the environment). Figures 10-12 give the same results in graphical form for three runs at 1500 RPM. As pointed out earlier, the greatest amount of negative heat transfer (from the cap to the gas) occurs when all convective heat transfer coefficient values during intake are increased by a factor of 5. This is shown in Figure 11.

All results shown in Figures 1-12 are for an aluminum cap. The properties of aluminum used for the simulation were 3.3 ft^2/hr for thermal diffusivity and 155 BTU/hr-ft-F for thermal conductivity. For comparison a benchmark run at 1500 RPM was made for a steel cap. Properties of the steel were 0.48 ft^2/hr and 26 BTU/hr-ft-F for thermal diffusivity and thermal conductivity respectively.

Figure 13 gives the transient response for steel at 1500 RPM with benchmark values. Comparisons with an aluminum cap at 1500 RPM (Figure 5) lead to the expected conclusions. The thermal oscillations in the steel are much more damped than in aluminum, and the time constant for steel is much larger than for aluminum. In fact, after 13 seconds the edge of the cap has exceeded 1000 F and is still increasing. Thus, the use of a steel cap increases its operating temperature by more than 300 F over most of

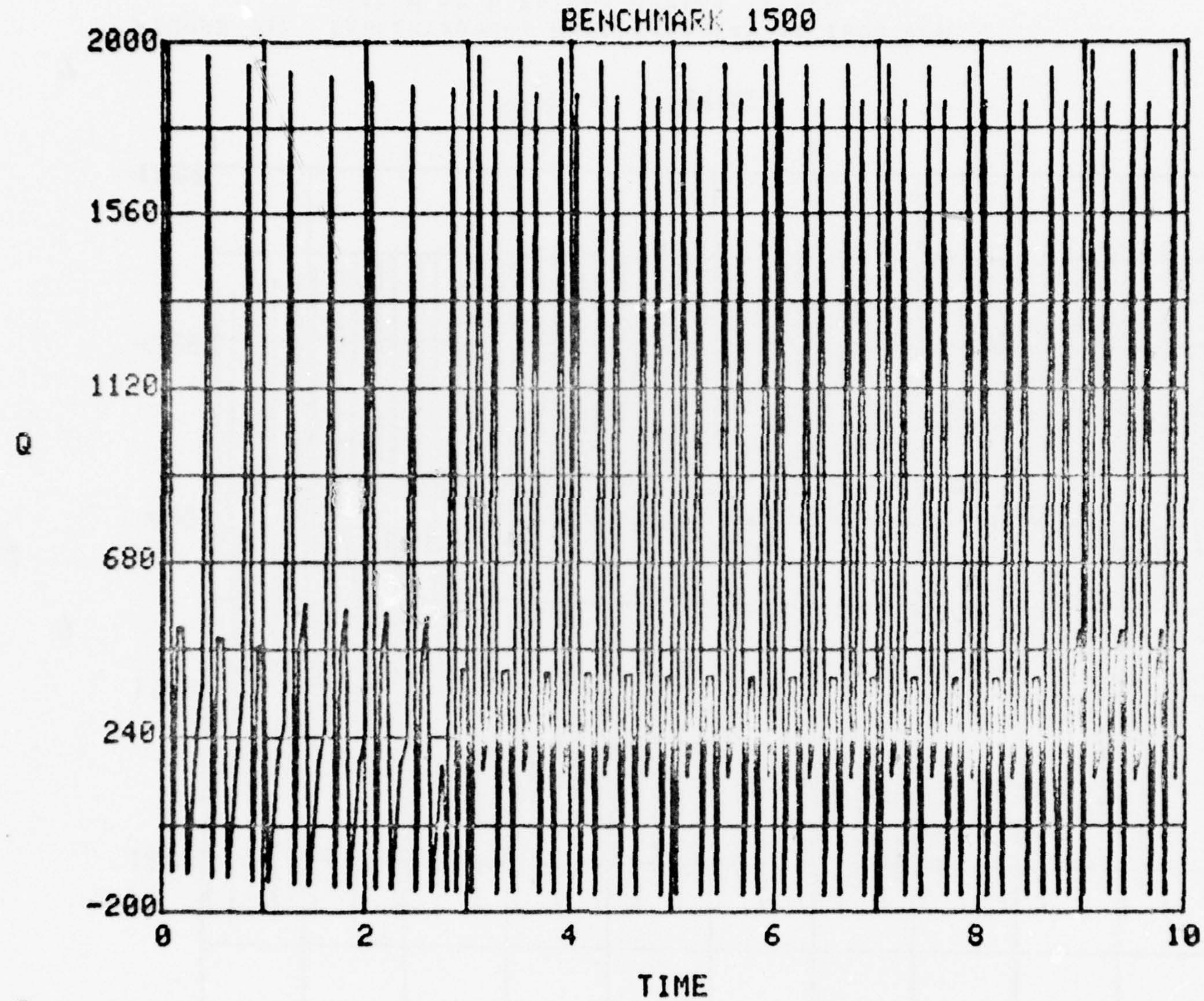

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FIGURE 10: Instantaneous Heat Transfer at 1500 RPM with Benchmark values

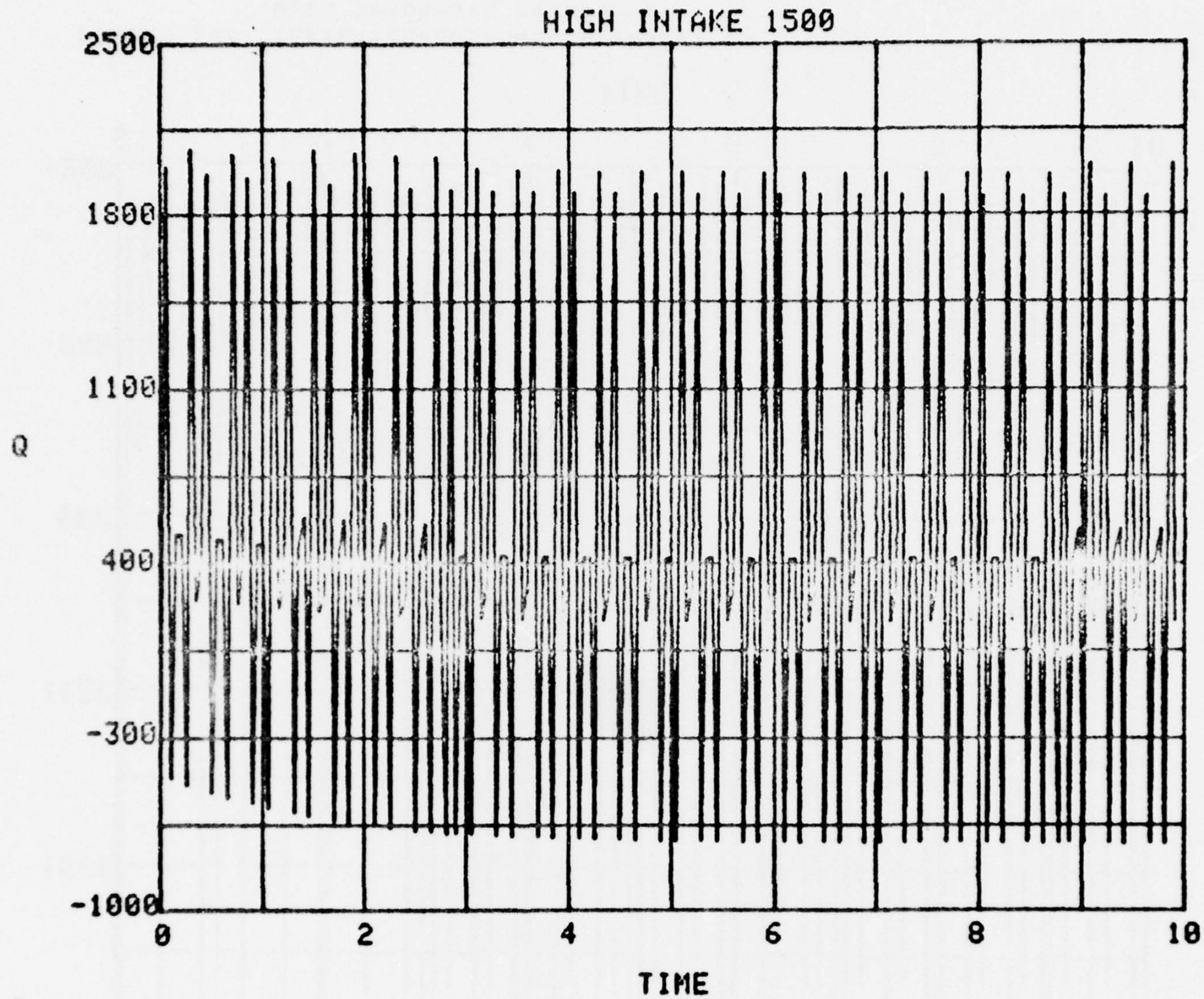

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FIGURE 11: Instantaneous Heat Transfer at 1500 RPM with High h Values during Intake

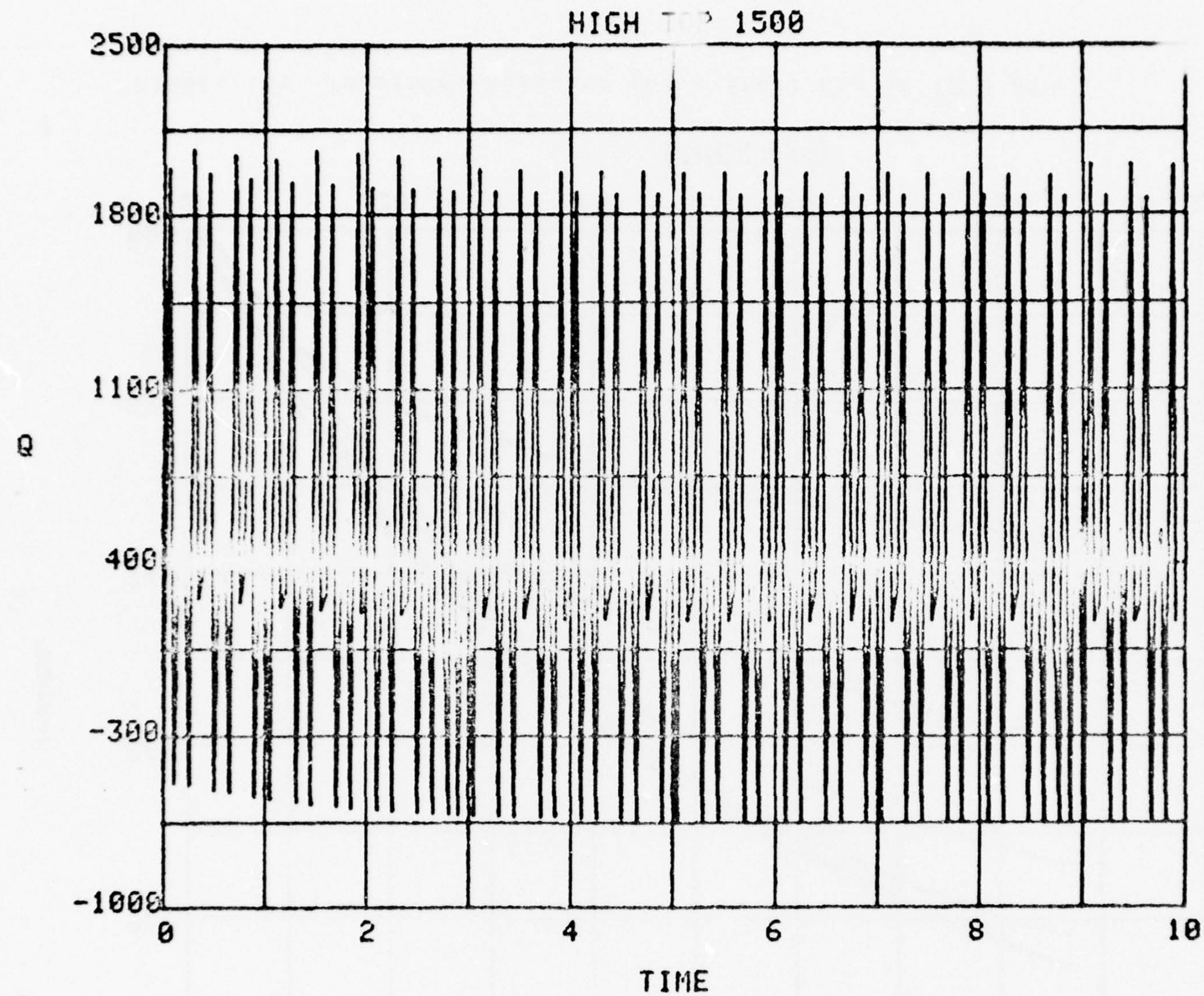

FIGURE 12: Instantaneous Heat Transfer at 1500 RPM with High h Value of Top Surface during Intake

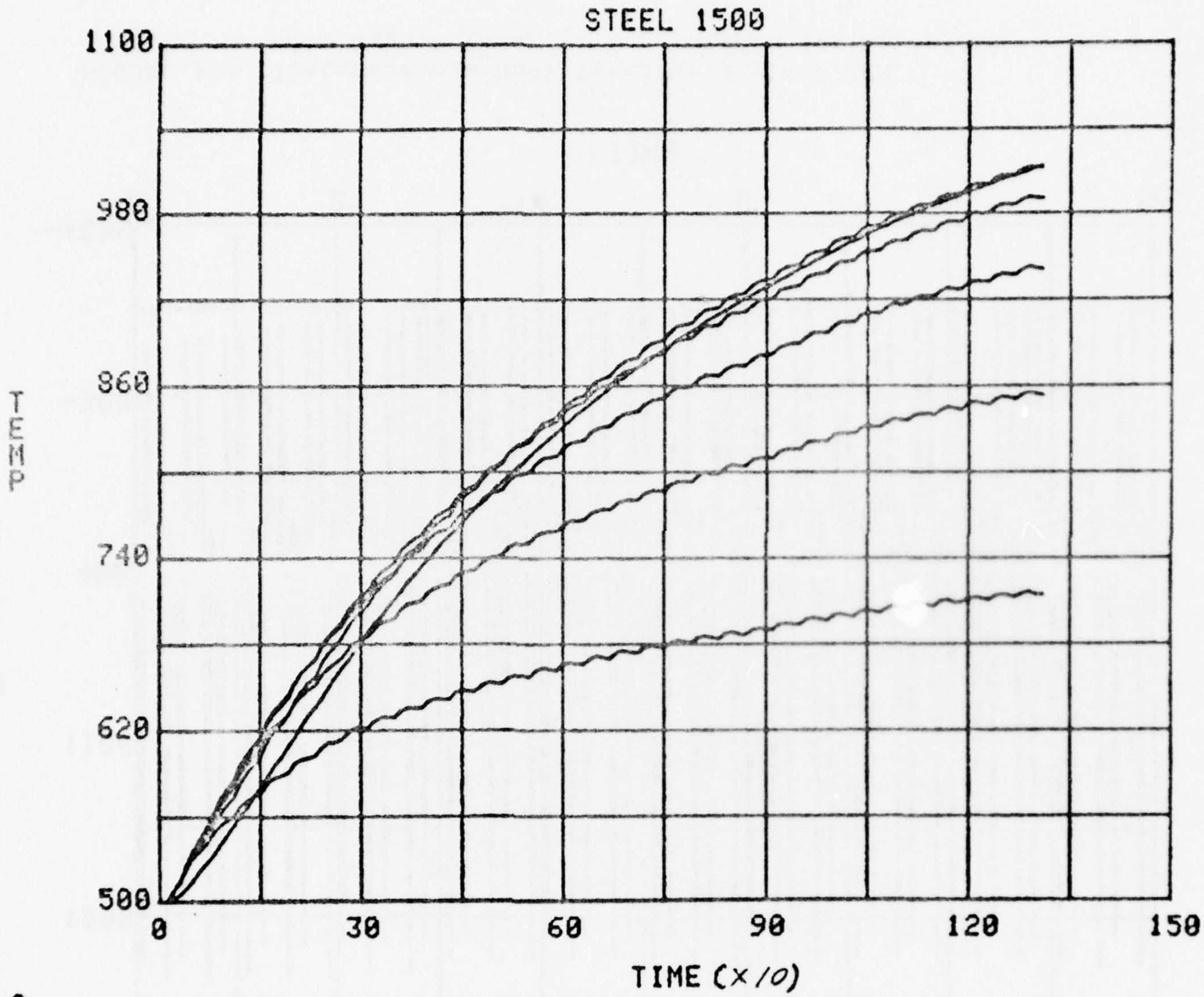

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FIGURE 13: Transient Response for a Steel Cap At 1500 RPM

the cap surface, as compared to an aluminum cap. The heat transfers to the gas from the cap during the first ten seconds at 1500 RPM is 0.1712 BTU for a steel cap as compared to 0.118 BTU for an aluminum cap. This increase is due to the increased operating temperature of the cap. Conversely, the heat transfer from the gas to the cap is 1.538 BTU for the steel and 1.728 BTU for the aluminum.

B. Eaton Series with Parametric Variations

Results for computer simulations using the convective heat transfer values given in TABLE B begin with Figure 14. Comparison of Figures 14 and 4 show that the fin reaches quasi-steady temperatures faster, and the predicted temperatures are much higher. Aluminum fin edge temperatures around 1,000 F are predicted by the model. The heat transfer to the gas from the fin is increased from 0.109 BTU to 2.63 BTU, a significant change!

At 1500 RPM the difference between the benchmark values and Easton values are shown in Figure 5 and 15. For this case the fin-gas heat transfer increases from 0.111 BTU to 2.87 BTU. Similar trends were noted at 5000 RPM. The corresponding instantaneous figures are Figures 6 and 16. The heat flux values for the benchmark runs (Figures 10-12) show much less negative heat transfer (from fin to gas) than in the Eaton runs (Figures 17-19). Of course the positive heat transfer is also greater.

When a steel fin is used with the Eaton convective heat transfer coefficient values, temperatures at the edge of the fin exceed 1200 F as shown in Figure 20. The heat transfer from the steel fin to the gas during the first ten seconds is 3.7 BTU.

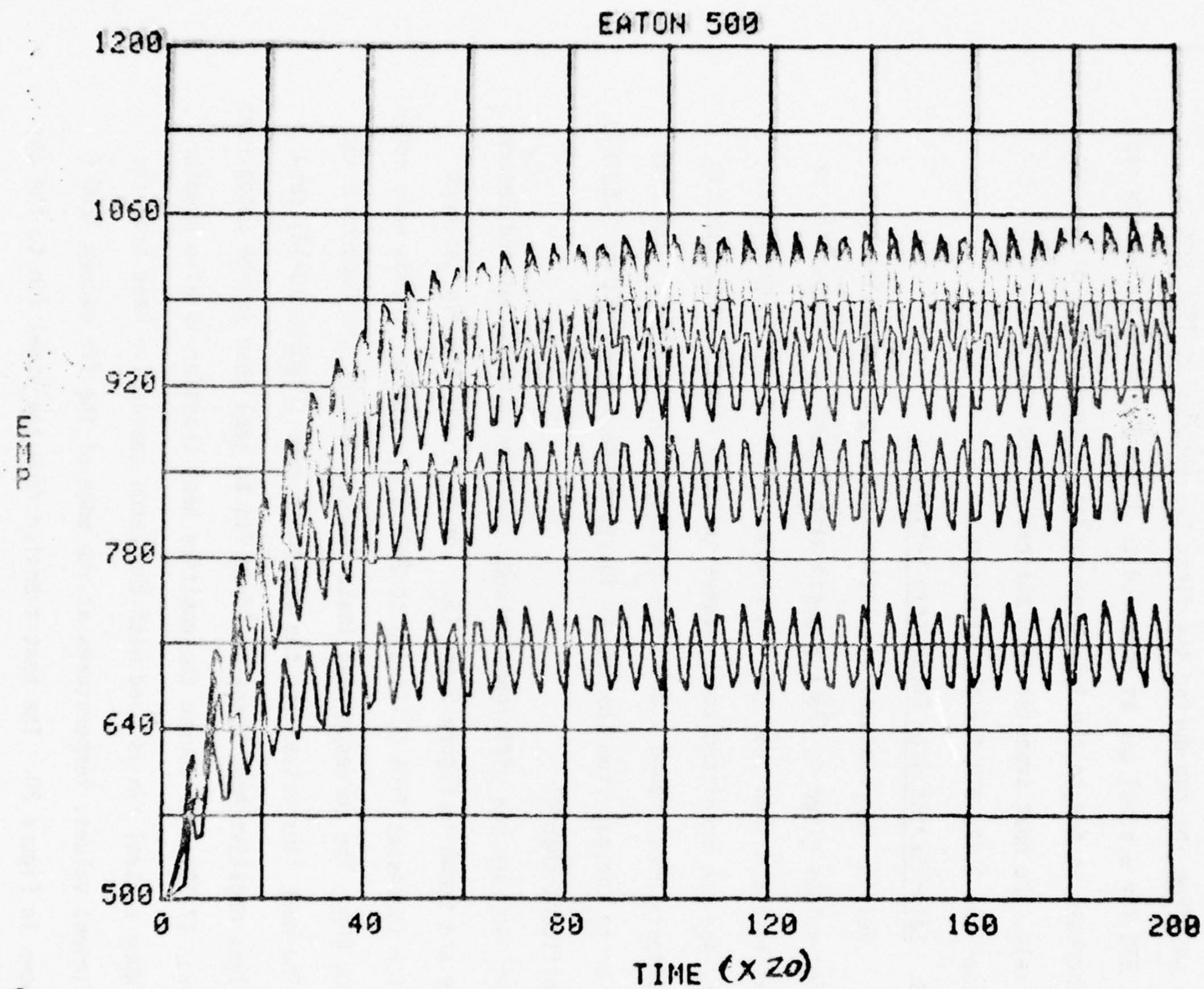

Figure 14: Transient Response at 500 RPM, Eaton Series

8.752	500	673.9	804.5	894.7	950.8	984.6	1002.5	HT= 869.052
8.804	500	725	855.7	943.8	1000.8	1030.6	1012.4	HT= 9700.15
8.852	500	740	884.5	973.8	1028.4	1049.1	1039.9	HT= 620.13
8.904	500	712	860.2	952.3	1005	1027.6	1040.6	HT=-3646.35
8.952	500	680.1	820.4	912.7	966.6	996.2	1020.9	HT=-1251.9
9.004	500	677.8	808.8	899.2	955.6	989.4	1004.8	HT= 1467.93
9.052	500	732.7	865.8	954.5	1011.6	1040.3	1019.6	HT= 9837.79
9.104	500	734.6	881.5	971.9	1026.2	1047.1	1043.5	HT= 381.107
9.152	500	705.3	852.9	945.5	998.7	1022.7	1039.4	HT=-3619.58
9.204	500	677.6	815.3	907.6	962.5	994.1	1017.1	HT=-618.497
9.252	500	683.7	813.9	904.1	961.2	995.3	1004.5	HT= 9135.21
9.304	500	745.5	882	970.9	1027.8	1054.1	1028.8	HT= 10044.4
9.352	500	731.7	879.4	970.6	1024.7	1046	1045	HT= 221.995
9.404	500	696.6	842.5	935.5	989	1014.9	1035.6	HT=-3578.94
9.452	500	676.8	812.9	905.1	960.6	993.1	1014.9	HT=-199.395
9.504	500	699.6	829.3	919.1	976.6	1010.4	1007.1	HT= 9344.23
9.552	500	748.2	887.3	976.4	1033.1	1057.7	1035.3	HT= 1012.87
9.604	500	727.4	876	967.9	1021.7	1043.6	1045.9	HT=-14.1353
9.652	500	691.2	835.9	929	982.8	1010	1032.7	HT=-3552.66
9.704	500	676.4	810.5	902.3	958.4	992	1011.6	HT= 426.626
9.752	500	709.7	839.8	929.4	987.1	1020	1010.6	HT= 9483.08
9.804	500	743	886	975.7	1031.7	1054.1	1041.3	HT= 772.314
9.852	500	722.4	871.3	963.6	1017.3	1039.7	1046.1	HT=-3698.27
9.904	500	683.7	826	919.1	973.4	1002.4	1027.5	HT=-3512.77
9.952	500	676.8	809.7	901.2	957.8	991.7	1009.9	HT= 841.152
10.004	500	723.9	855.8	945.1	1002.8	1033.6	1017.1	HT= 9691.88

TOTAL Q= 4.95338 AFTER 10.004 SEC   TIME INC= 0.004

NEG Q=-2.62586 POS Q= 7.57924

TABLE 7: Numerical Values, Eaton Series, RPM = 500

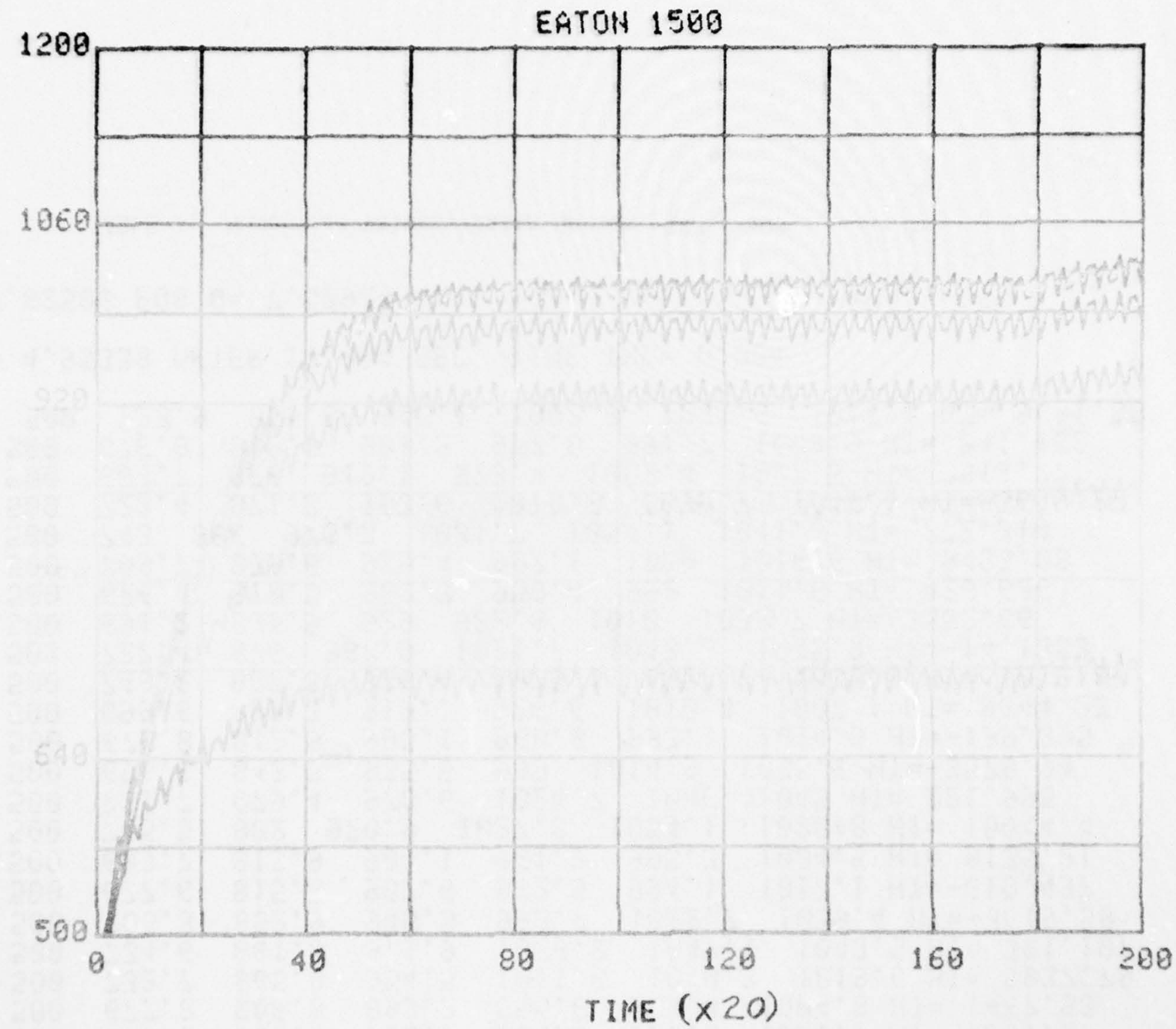

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Figure 15: Transient Response at 1500 RPM, Eaton Series

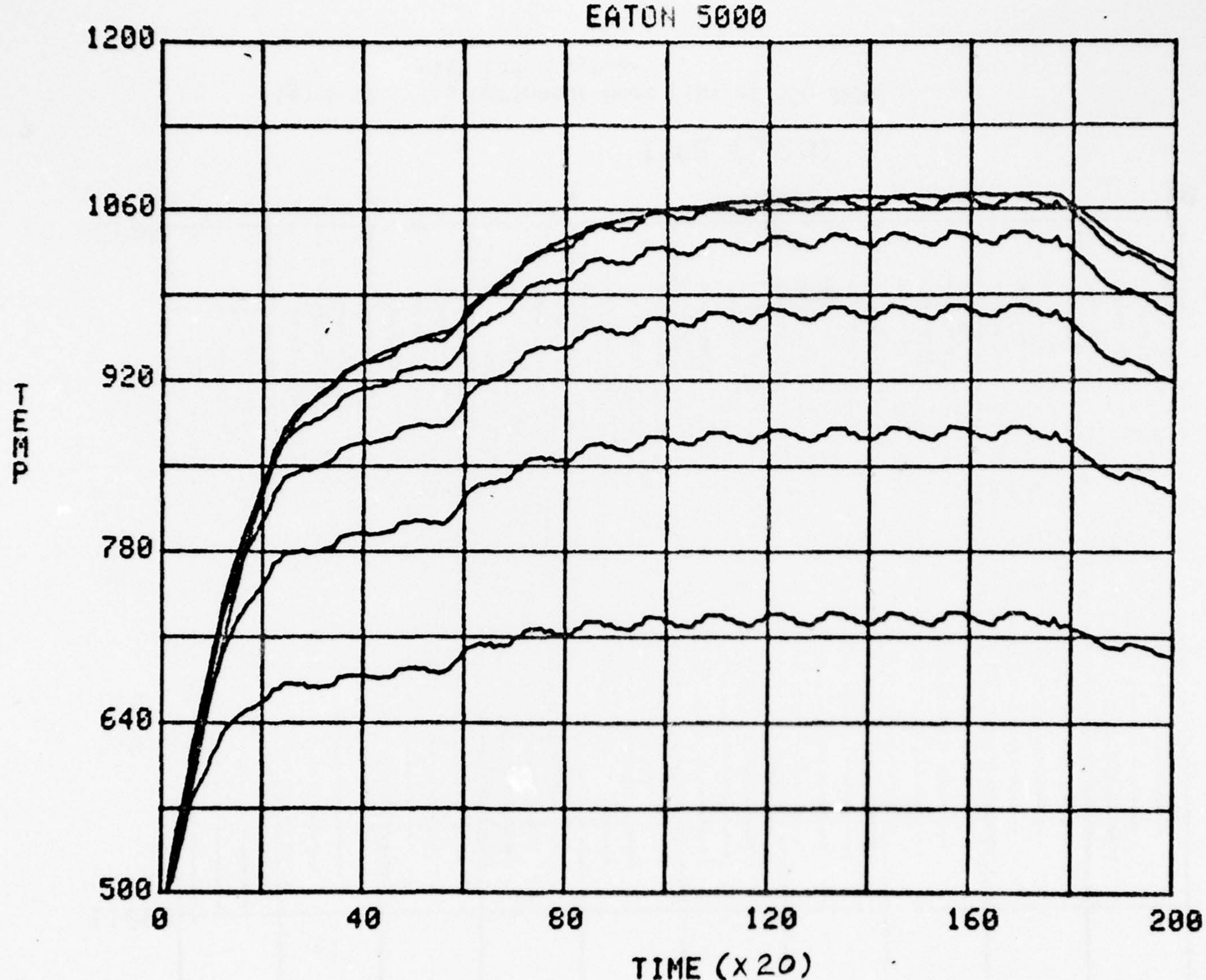

Figure 16: Transient Response to 5000 RPM, Eaton Series

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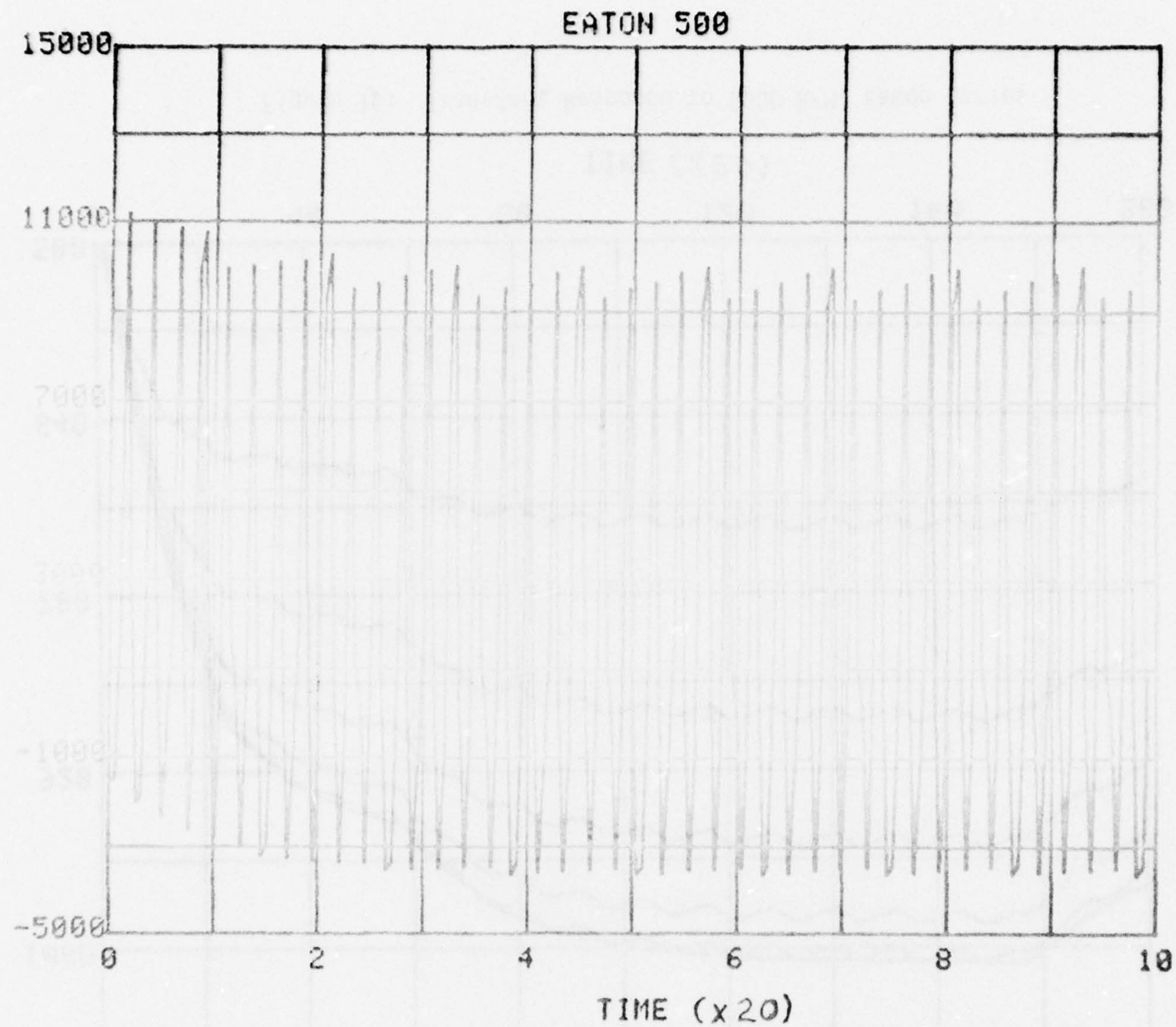

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Figure 17: Instantaneous Heat Flux at 500 RPM with Eaton Valves

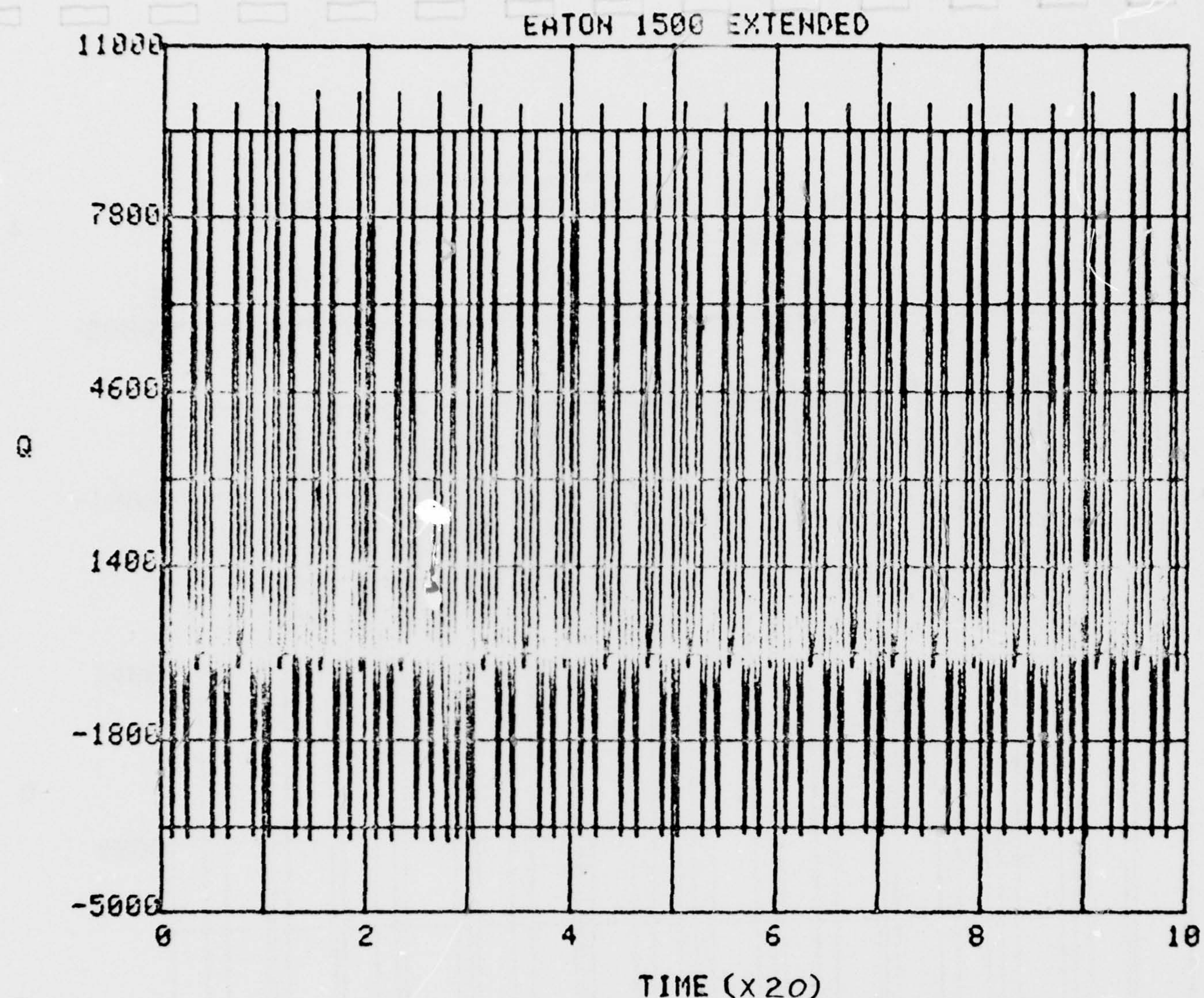

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Figure 18: Instantaneous Heat Flux at 1500 RPM with Eaton Valves

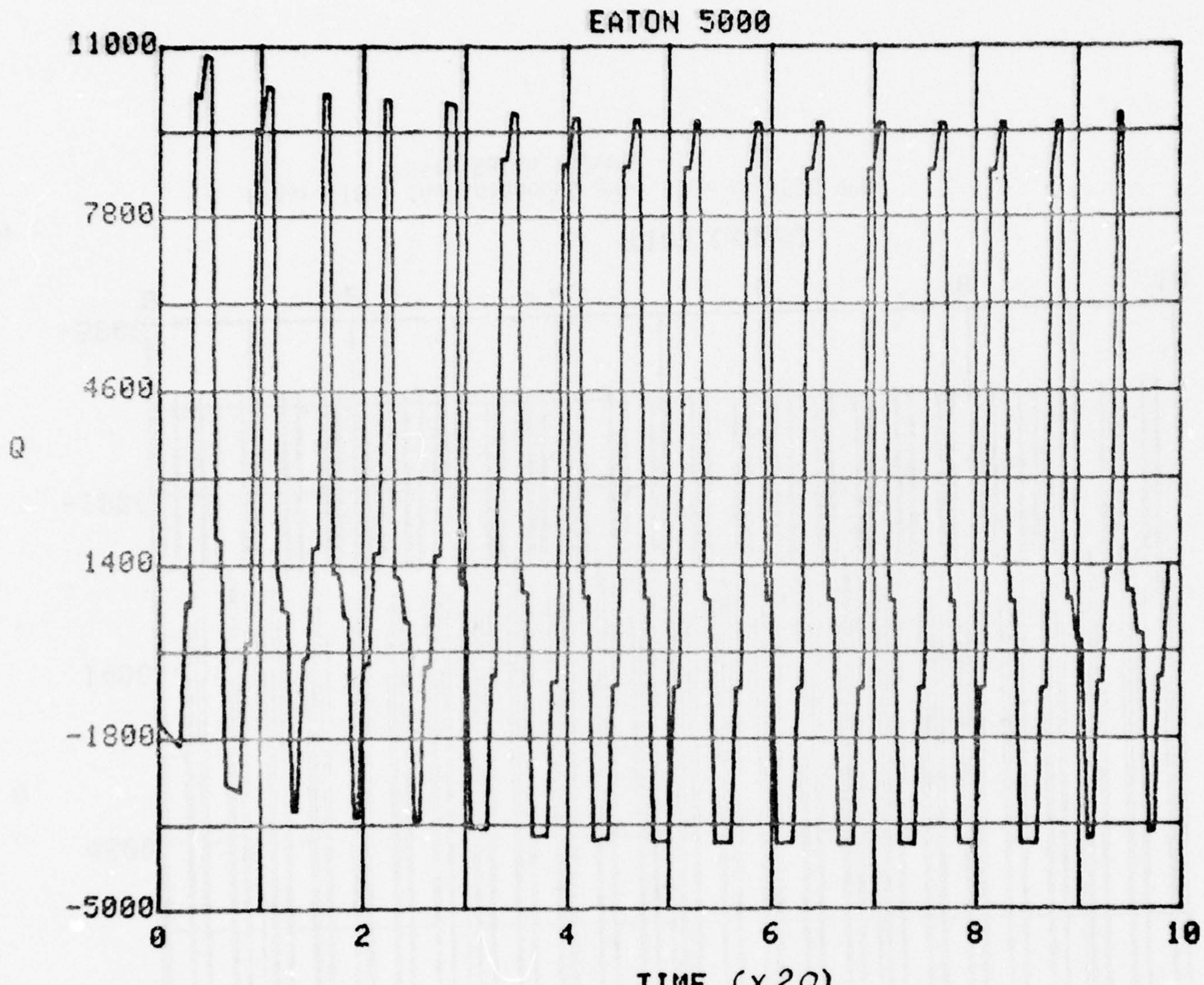

Figure 19: Instantaneous Heat Flux at 5000 RPM with Eaton Valves

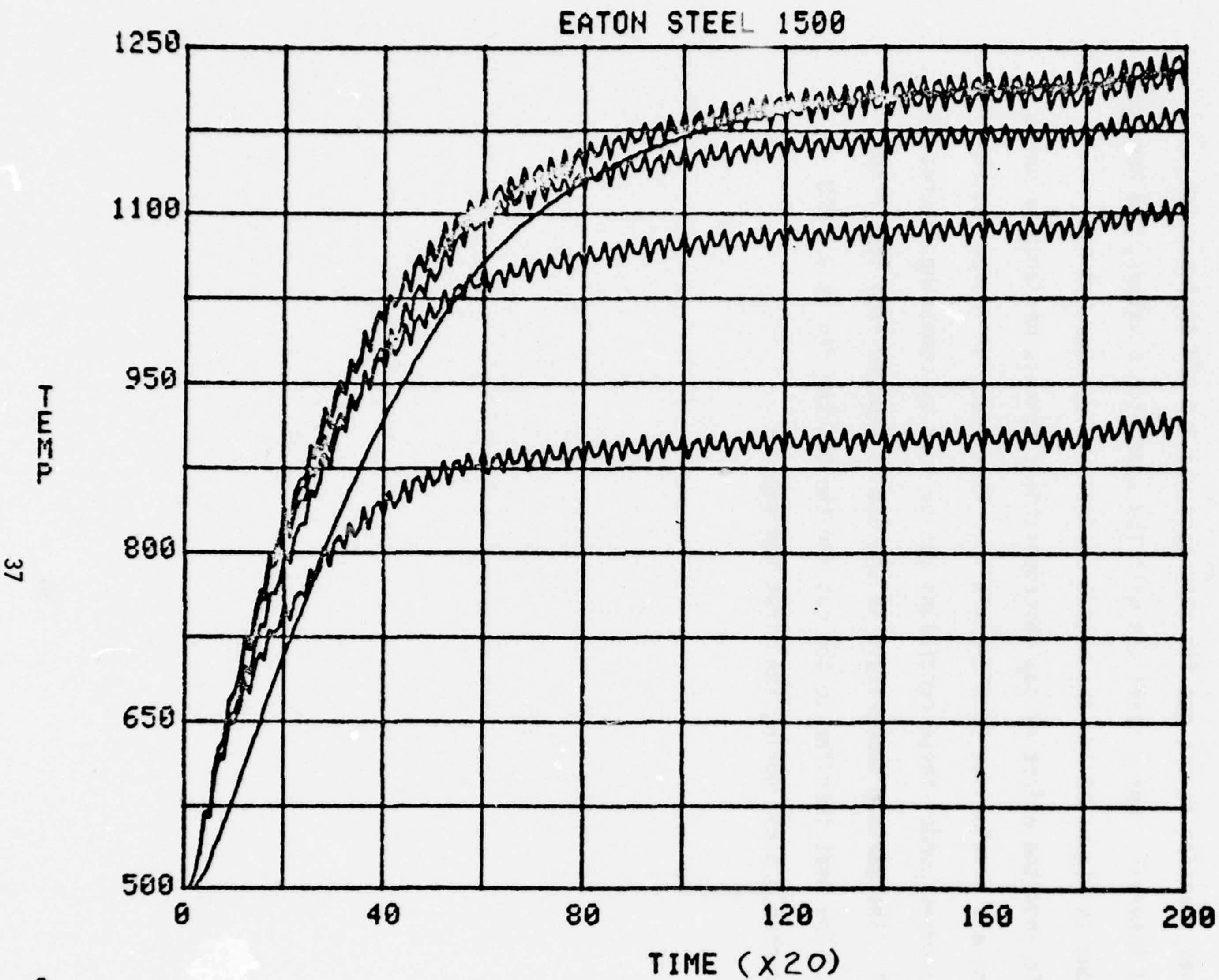

Figure 20: Steel Cap at 1500 RPM, Eaton Valves

For a parametric change to the Eaton values of h, all values of h were increased, to 80 BTU/hr-ft^2-F. The results for 1500 RPM are shown in Figure 21 for an aluminum cap and should be compared with Figure 15. The heat transfer to the gas for this case was 3.9 BTU during a ten second interval. For a steel cap with the same high h values, the response is shown in Figure 22 and the heat transfer obtained was 5.4 BTU.

To show the effect of cap thickness the thickness was doubled while keeping all h values at 80 BTU/hr-ft^2-F. The effect of increasing the cap thickness under these conditions can be seen by comparing Figures 23 and 21. The maximum temperature in the cap is reduced from 1096 F to 958 F. The heat transfer to the gas for the thicker fin is 3.0 BTU rather than 3.8 BTU during the first ten seconds.

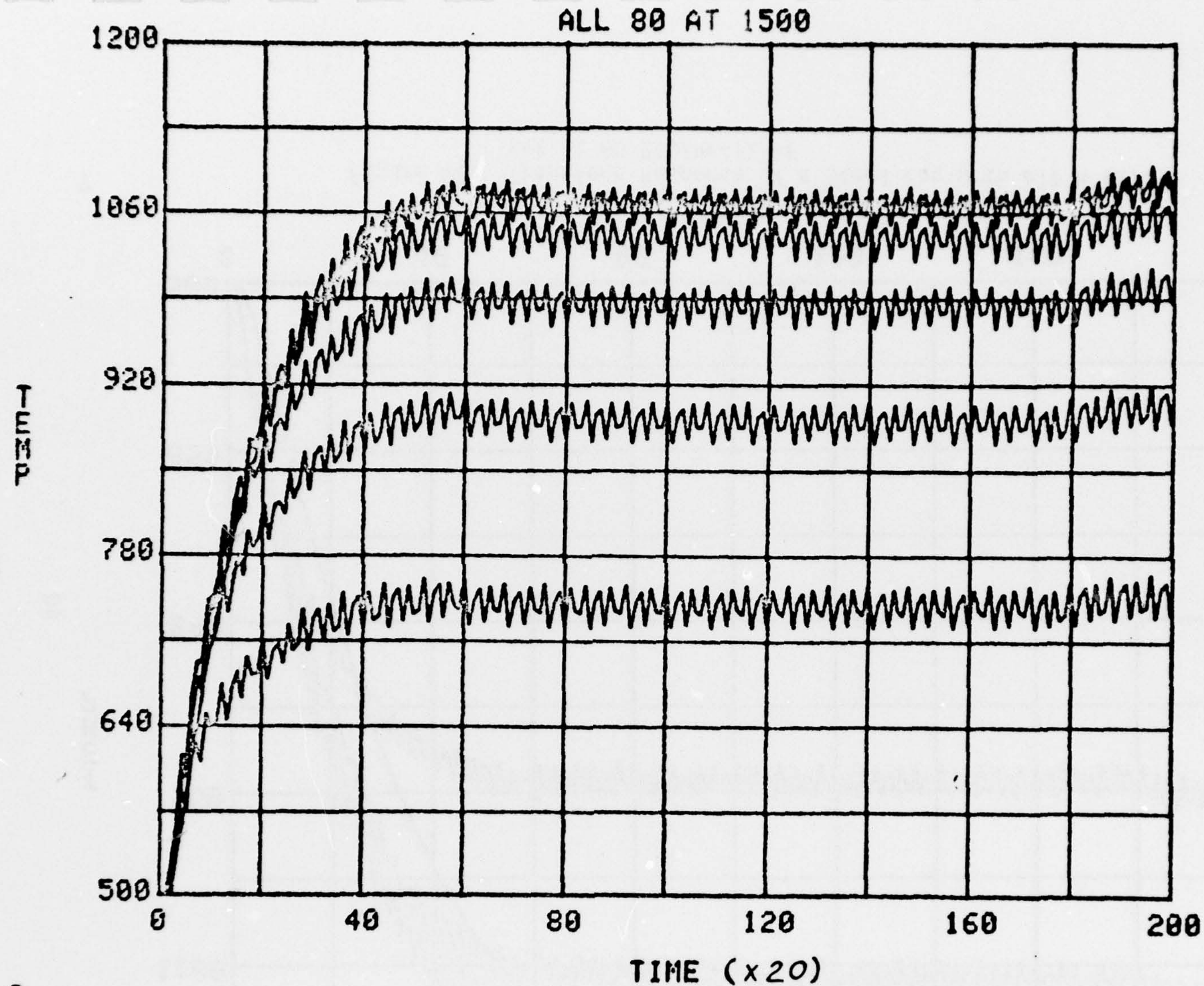

Figure 21: Transient Response of Aluminum Cap with all h valves set at 80 BTU/hr-ft^2-F

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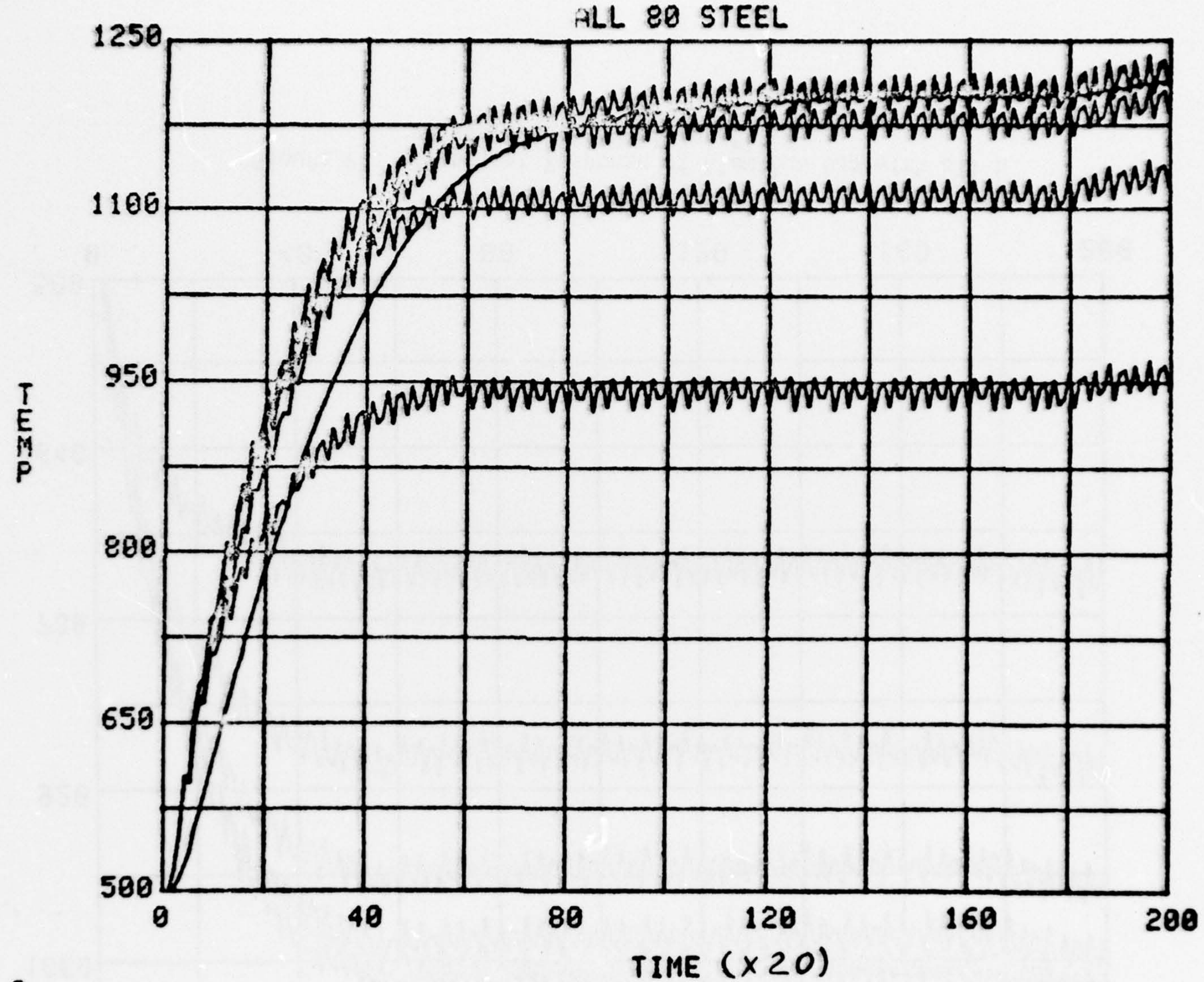

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Figure 22: Transient Response of a Steel Cap with all h valves set at 80 BTU/hr-ft^2-F

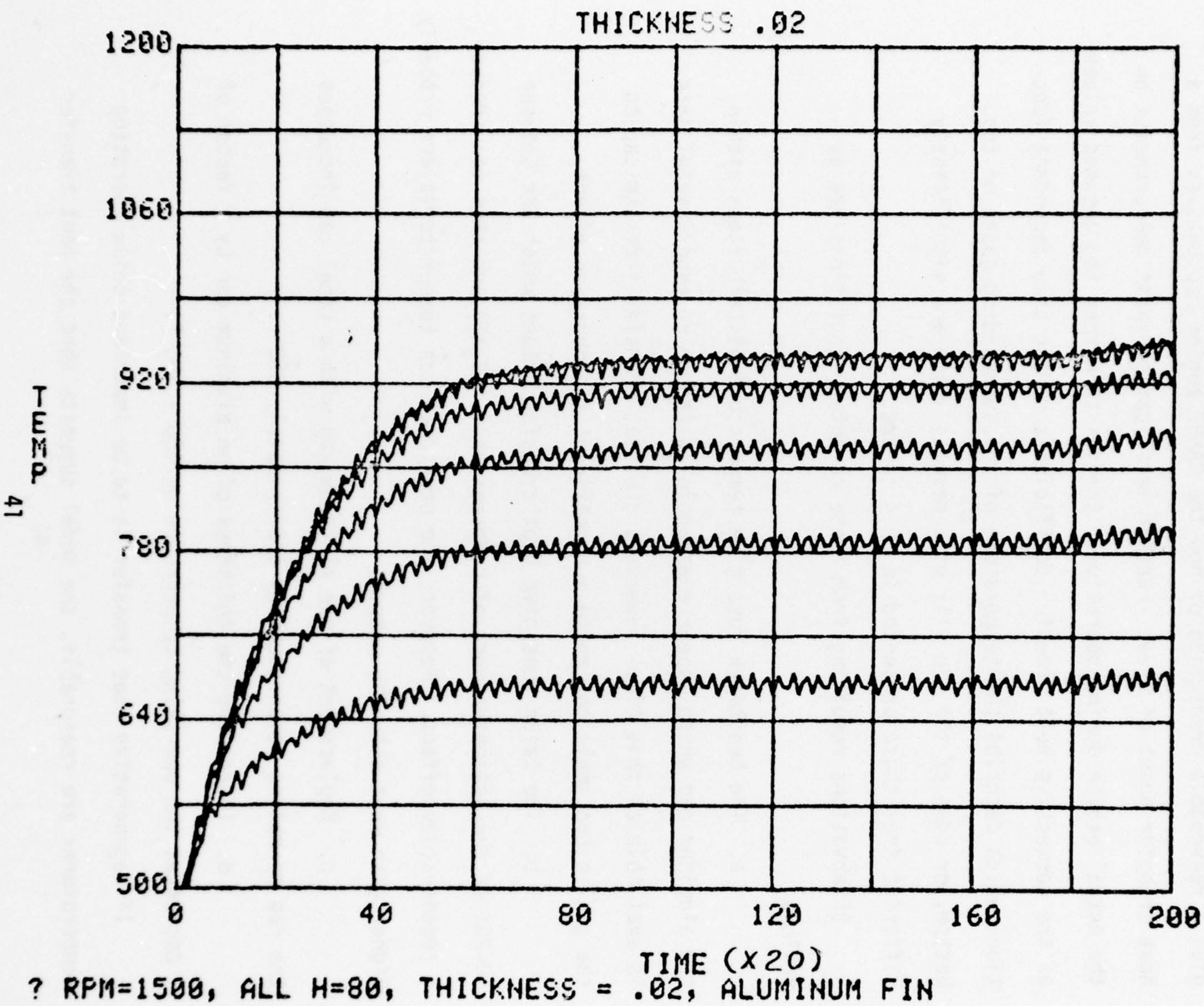

Figure 23: Transient Response for a Thick Cap

V CONCLUSIONS

The mathematical model and resulting computer simulation appear to give a reasonable prediction of how the NAHBE engine cap behaves from a heat transfer point of view. Further verification await measurements on the actual engine during operation. Changes to either the assumed values of the convective heat transfer coefficients or the time dependent functions which describe the temperature of the surrounding gases on top, bottom, or edge of the cap will give results which are significantly different from those presented in this report.

Observations resulting from these computer simulations are as follows:

a. The benchmark runs give temperature distributions within the aluminum cap which appear reasonable, although no experimental data is available at this time. However, the heat transfer from the cap to the gas is too small to create a significant regenerative effect.

b. The Eaton runs give heat transfer values which are between 10-20% of the estimated work of compression ($\dot{m}\, C_V \Delta T$) and thus can produce a regenerative effect. However, the predicted cap temperatures are probably higher than what actually occurs.

c. Replacement of the aluminum cap with a steel cap increases the cap operating temperature by approximately 300 F.

d. Increasing the thickness of an aluminum cap by a factor of 2 decreases the operating temperature by 100 - 150 F.

If regenerative heat transfer is to be important while operating temperatures are reasonable, the model suggests that the heat transfer

coefficient (e.g. non-dimensional gas temperature gradient at the cap surface) must be low when the environmental gas temperature is high (to reduce heat transfer to the cap), and high when the environmental gas temperature is lower than the cap temperature (to increase heat transfer from the cap). One hypothesis is that during compression the filling of the cavity beneath the cap prevents the build up of a "thick" boundary layer on the top cap surface and this produces a much higher heat transfer from the cap to the gas during compression. Also, it is possible that during the complicated combustion process, gradients may be set up in the ionized gases to effectively reduce the heat transfer coefficient, or the effective temperature difference between surface and gas, during the combustion/expansion stroke.

REFERENCES

1. Blaser, R., Pouring, A., Keating, E., and Rankin, B., "The Naval Academy Heat Balance Engine," USNA Report EW 8-76, June, 1976.

2. Pouring, A. A., et al, "The Influence of Combustion with Pressure Exchange on the Performance of Heat Balanced Internal Combustion Engines," paper 770120, Society of Automotive Engineers, February 28, 1977.

3. Davis, L. J. "Engine Valve Cooling for Emission Control," Paper 730055, presented at The Society of Automotive Engineers, January 8-18, 1973.

VI

APPENDIX A

COMPUTER PROGRAM LISTINGS

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        11:25:39            08/30/77
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```
100 LIBRARY"INITIAL","EXP","OUT"
110 FILE #1:"TRANS"
120 FILE #2:"HEAT"
130 FILE #3:"INIT"
140 SCRATCH #1
150 SCRATCH #2
160 DIM O(3000)
170 LET O1=1
180 DIM T(20),F(20)
190 DIM P(20)
200 REM INPUT CAP THICKNESS(IN)
210 LET D=0.01
220 REM INPUT CAP LENGTH(IN)
230 LET L=0.75
240 REM INPUT NO OF NODES
250 LET N=13
260 PRINT"INPUT DELTA TIME";
270 INPUT I0
280 LET X=L/(N-1)
290 PRINT
300 REM INPUT THERMAL DIFFUSIVITY(FT^2/HR)
310 LET A=3.3
320 REM INPUT THERMAL CONDUCTIVITY(BTU/HR-FT-F)
330 LET K=155
340 LET F=A*I0*144/((X^2)*3600)
350 PRINT
360 PRINT"INPUT PRINT INTERVAL(SEC), MAX TIME(SEC)";
370 INPUT P,E
380 PRINT
390 REM ASSIGN INITIAL TEMP DISTRIBUTION
400 CALL"INITIAL":N,L,K,P(),T(),#3
410 LET P1=P
420 REM APLY EXPLICIT FINITE DIFFERENCE EQNS
430 CALL"EXP":D,N,L,A,K,F,P,E,T(),O(),O1,H1,H2,H9,I0,#2,#3
440 REM PUT TRANSIENT RESPONSE ON FILE #1
450 LET Z9=INT(N/2)+1
460 LET Z8=INT(E/P1)
470 FOR I=1 TO Z9
480 FOR J=1 TO Z8+1
490 PRINT #1:J;",";O((J-1)*Z9+I)
500 NEXT J
510 PRINT #1:"1E37,1E37"
520 NEXT I
530 PRINT #2:"1E37,1E37"
540 END
```

```
100 SUB"INITIAL":N,L,K,P(),T(),#3
110 MAT T=ZER(N+1)
120 LET X=0
130 FOR I=1 TO 13
140 INPUT #3:P(I)
150 NEXT I
160 FOR I=1 TO N
170 LET T(I)=P(I)
180 LET X=X+L/(N-1)
190 NEXT I
200 SUBEND
```

```
100 SUB"EXP":D,N,L,A,K,F,P,E,T(),O(),O1,H1,H2,H9,I0,#2,#3
110 REM DEF H ON TOP
120 DEF FNX(Z0)
130 IF Z0<X1 THEN 180
140 IF Z0<X2 THEN 200
150 IF Z0<X4 THEN 220
160 LET FNX=10
170 GO TO 230
180 LET FNX=25
190 GO TO 230
200 LET FNX=10
210 GO TO 230
220 LET FNX=10
230 FNEND
240 DEF FNB(Z0)
250 REM DEF T GAS ON BOTTOM
260 IF Z0<X1 THEN 350
270 IF Z0<X2 THEN 310
280 IF Z0<X4 THEN 330
290 LET FNB=((W5-W4)/(X5-X4))*(Z0-X4)+W4
300 GO TO 360
310 LET FNB=((W2-W1)/(X2-X1))*(Z0-X1)+W1
320 GO TO 360
330 LET FNB=((W4-W3)/(X4-X3))*(Z0-X3)+W3
340 GO TO 360
350 LET FNB=W0
360 FNEND
370 DEF FNT(Z0)
380 REM DEF T GAS ON TOP
390 IF Z0<X1 THEN 480
400 IF Z0<X2 THEN 440
410 IF Z0<X4 THEN 460
420 LET FNT=Y2
430 GO TO 490
440 LET FNT=((Y2-Y1)/(X2-X1))*(Z0-X1)+Y1
450 GO TO 490
460 LET FNT=((Y4-Y3)/(X4-X3))*(Z0-X3)+Y3
470 GO TO 490
480 LET FNT=Y0
490 FNEND
500 DEF FNY(Z0)
510 REM DEF H ON BOTTOM
520 IF Z0<X1 THEN 570
530 IF Z0<X2 THEN 590
540 IF Z0<X4 THEN 610
550 LET FNY=10
560 GO TO 620
570 LET FNY=5
580 GO TO 620
590 LET FNY=10
600 GO TO 620
```

```
610 LET FNY=10
620 FNEND
630 DEF FNE(Z0)
640 REM DEF T GAS ON EDGE
650 IF Z0<X1 THEN 700
660 IF Z0<X2 THEN 720
670 IF Z0<X4 THEN 740
680 LET FNE=1000
690 GO TO 750
700 LET FNE=530
710 GO TO 750
720 LET FNE=300
730 GO TO 750
740 LET FNE=1218
750 FNEND
760 DEF FNZ(Z0)
770 REM DEF H ON EDGE
780 IF Z0<X1 THEN 830
790 IF Z0<X2 THEN 850
800 IF Z0<X4 THEN 870
810 LET FNZ=10
820 GO TO 880
830 LET FNZ=5
840 GO TO 880
850 LET FNZ=10
860 GO TO 880
870 LET FNZ=10
880 FNEND
890 REM INPUT RPM
900 LET N5=500
910 LET X9=60/N5
920 REM DEF X-Y FOR T TOP
930 LET X0=0
940 LET Y0=70
950 LET X1=0.5*X9
960 LET Y1=530
970 LET X2=X9
980 LET Y2=1218
990 LET X3=X2
1000 LET Y3=4000
1010 LET X4=3*X9/2
1020 LET Y4=4500
1030 LET X5=2*X9
1040 LET Y5=Y2
1050 REM DEF Y FOR T BOTTOM
1060 LET W0=530
1070 LET W1=530
1080 LET W2=1218
1090 LET W3=1218
1100 LET W4=1218
1110 LET W5=W1
```

```
1120 DIM U(30),X(20),R(20)
1130 DIM Q(20)
1140 REM CALC DELTA R
1150 LET X6=L/(N-1)
1160 REM CALCULATE INITIAL VALUES
1170 LET Z=0
1180 LET Z0=0
1190 LET H1=FNX(Z0)
1200 LET H2=FNY(Z0)
1210 LET H9=FNZ(Z0)
1220 LET S9=(3.14159*(1.75^2-1)/144)*(H1*(FNT(Z0)-T(1))+H2*(FNB(Z0)-T(1)))+(H9*3.14159*3.25*0.01/144)*(FNT(Z0)-T(1))
1230 CALL"OUT":N,L,A,K,Z,T(),O(),O1,S9,#2
1240 REM APPLY FINITE DIFFERENCE EQNS
1250 LET C=P
1260 LET U5=A*I0*12/(K*D*3600)
1270 LET U(1)=T(1)
1280 LET B=H9*X6/(12*K)
1290 LET Q9=0
1300 LET P9=0
1310 LET N9=0
1320 LET H1=FNX(Z0)
1330 LET H2=FNY(Z0)
1340 LET H9=FNZ(Z0)
1350 LET R(1)=1
1360 FOR I=2 TO N-1
1370 LET R(I)=1+(I-1)*X6
1380 LET U(I)=T(I)*(1-U5*(H1+H2)-2*F)+F*(T(I+1)+T(I-1))+(F/2)*(X6/R(I))*(T(I+1)-T(I-1))+U5*(H1*FNT(Z0)+H2*FNB(Z0))
1390 LET Q(I)=(3.14159*(R(I)^2-R(I-1)^2)/144)*(H1*(FNT(Z0)-U(I))+H2*(FNB(Z0)-U(I)))
1400 NEXT I
1410 LET I=I+1
1420 LET U(I)=F*(T(I-1)+B*FNE(Z0))+(1-F*(1+B))*T(I)
1430 LET Q(I)=(3.14159*(1.75^2-R(N-1)^2)/144)*(H1*(FNT(Z0)-U(I))+H2*(FNB(Z0)-U(I)))+H9*(3.14159*3.25*0.01/144)*(FNE(Z0)  (I
1440 LET S9=0
1450 FOR I=2 TO N
1460 LET S9=S9+Q(I)
1470 NEXT I
1480 IF S9<0 THEN 1510
1490 LET P9=P9+S9*I0/3600
1500 GO TO 1520
1510 LET N9=N9+S9*I0/3600
1520 LET Q9=Q9+S9*I0/3600
1530 LET Z=Z+I0
1540 LET I5=INT(Z/(2*X9))
1550 LET Z0=Z-I5*2*X9
1560 IF Z<C THEN 1600
1570 CALL"OUT":N,L,A,K,Z,T(),O(),O1,S9,#2
1580 LET C=C+P
1590 IF Z>E THEN 1640
1600 FOR I=2 TO N
1610 LET T(I)=U(I)
1620 NEXT I
```

```
1630 GO TO 1320
1640 PRINT
1650 PRINT"TOTAL Q=";Q9;"AFTER";Z;"SEC";"  TIME INC=";I0
1660 PRINT
1670 PRINT"NEG Q=";N9;"POS Q=";P9
1680 SCRATCH #3
1690 FOR I=1 TO 13
1700 PRINT #3:U(I)
1710 NEXT I
1720 SUBEND
```

```
100 SUB"OUT":N,L,A,K,Z,T(),O(),O1,S9,#2
110 DIM D(20),F(20),Q(20),X(20)
120'CALCULATE AND PRINT TIME
130 LET T=Z
140 LET T=INT(T*10^3+0.5)/10^3
150 PRINT T;
160'CALCULATE DIMENSIONAL TEMPERATURE
170 FOR I=1 TO N
180 LET D(I)=T(I)
190 LET F(I)=INT(D(I)*10+0.5)/10
200 NEXT I
210'PRINT ODD NODAL TEMPERATURES
220 FOR I=1 TO N STEP 2
230 PRINT F(I);
240 LET O(O1)=F(I)
250 LET O1=O1+1
260 NEXT I
270 PRINT"HT=";S9
280 PRINT #2:Z,",",S9
290'CALCULATE SURFACE HEAT FLUX
300 LET Q(1)=K*(D(2)-D(1))/(L/(N-1))
310 LET X(1)=0
320'CALCULATE INTERNAL HEAT FLUXES
330 FOR I=2 TO N-1
340 LET Q(I)=K*(D(I+1)-D(I-1))/(2*L/(N-1))
350 LET X(I)=X(I-1)+L/(N-1)
360 NEXT I
370 LET Q(N)=0
380 LET X(N)=L
390'PRINT HEAT FLUX VALUES IF DESIRED
400 REM PRINT LOOP
410 SUBEND
```